QUESTION 1. A.

\[ Q(m) = T(T+2) \sum_{j=1}^{m} \frac{\hat{\rho}_j^2}{T-j} = 464(466) \sum_{j=1}^{3} \frac{\hat{\rho}_j^2}{T-j} = 75.08 \]

B.

\[ H_0 : \text{There exists NO serial correlation} \]
\[ H_1 : \text{There exists serial correlation} \]

\[ \text{Test Statistics } Q(m)=75.08 > \chi^2_{2,0.05} = 5.9 \text{ (table value)} \]

Reject null hypothesis. That is, there exists significant serial correlation.

QUESTION 2.

\[ X_t = \phi X_{t-2} + Z_t - \theta Z_{t-3}; \quad Z_t \sim WN(0, \sigma^2) \]

A. If the series is invertible:

\[ (1 - \phi B^2)X_t = (1 - \theta B^3)Z_t \]

\[ \Phi(B) = 1, \text{ the roots lie outside unit circle.} \]

The condition for having \(|B| > 1\) is: \(-1 < \theta < 1\)

B. If the series is stationary:

\[ (1 - \phi B^2) = 0 \implies 1 = \theta B^3 \implies B = \frac{1}{\sqrt{\phi}} \]

The condition for having \(|B| > 1\) is: \(1 < \sqrt{\phi} < 1\)

C. Forecasts for \(k=3\) steps

\[ \hat{X}_n(k) = E[X_{n+k} | F_n] = E[X_{n+k} | X_n, X_{n-1}, ..., X_1] \]
\[ e_n(k) = X_{n+k} - \hat{X}_n(k) \]

\(k=1\)

\[ X_{n+1} = \phi X_{n-1} + Z_{n+1} - \theta Z_{n-2} \]

\(k=2\)

\[ X_{n+2} = \phi X_{n} + Z_{n+2} - \theta Z_{n-1} \]

\(k=3\)

\[ X_{n+3} = \phi X_{n+1} + Z_{n+3} - \theta Z_{n} \]

\(k=1\)

\[ X_n(1) = E[X_{n+1} | F_n] = E[\phi X_{n-1} + Z_{n+1} - \theta Z_{n-2} | F_n] \]
\[ X_n(1) = \phi X_{n-1} - \theta Z_{n-2} \]
\[ e_n(1) = [\phi X_{n-1} - \theta Z_{n-2}] - [\phi X_{n-1} + Z_{n+1} - \theta Z_{n-2}] = Z_{n+1} \]
k=2
\[ X_n(2) = E[X_{n+2} | F_n] = E[\phi X_n + Z_{n+1} - \theta Z_{n-1} | F_n] \]
\[ X_n(2) = \phi X_n - \theta Z_{n-1} \]
\[ e_n(2) = [\phi X_n - \theta Z_{n-1}] - [\phi X_n + Z_{n+1} - \theta Z_{n-1}] = Z_{n+1} \]

k=3
\[ X_n(3) = E[X_{n+3} | F_n] = E[\phi X_{n+1} + Z_{n+3} - \theta Z_n | F_n] \]
\[ X_n(3) = \phi X(1) - \theta Z_n \]
\[ e_n(3) = [\phi(\phi X_{n-1} - \theta Z_{n-2}) - \theta Z_n] - [\phi X_{n+1} + Z_{n+3} - \theta Z_n] \]
\[ e_n(3) = \phi^2 X_{n-1} - \phi \theta Z_{n-2} - \theta Z_n - \phi X_{n+1} - Z_{n+3} + \theta Z_n \]
\[ e_n(3) = \phi(\phi X_{n-1} - X_{n+1}) - \phi \theta Z_{n-2} - Z_{n+3} \]

QUESTION 3. PART A.
Trend stationary model  \( X_t = a + bt + Z_t; \quad Z_t : WN(0, \sigma^2) \)
Random walk with drift  \( X_t = a + X_{t-1} + Z_t; \quad Z_t : WN(0, \sigma^2) \)
Random walk model is I(1), i.e. the first difference gives a stationary process.
Whereas, a trend stationary model is the composition of I(0) and trend.
The graphs of both models look similar; not easy to distinguish.

PART B. Taking the logarithms of a series, eliminates the exponential growth,
linearizes a multiplicative model, transforms the data into continuous form.

PART C. Auto correlation(ACF) and Partial Autocorrelation (PACF) functions are
the tools to determine the order of the process. It gives information on
1. if there exists a trend in the model (as ACF will have a significant autocorrelation
which does not converge to zero)
2. if there exists seasonal movements in the model (as the ACF and PACF will have
the same wavelike pattern as the series. Differencing will not eliminate this pattern).
3. if the series is pure random (as the correlation coefficients will fluctuate within
\[ \pm \frac{2}{\sqrt{n}} \]; 95% confidence band.
The Q- statistics will inform us about the significance of the serial correlation.
4. The orders of AR and MA part of the series.

QUESTION 4. PART A. Cointegration analysis explains the long and short run
relationships of more than one series to another and specifies if the change in one
effect the others. These series are I(d) stationary. The linear relationship of k-
inTEGRATED MODELS OF ORDER d will result in a stationary series having an order less
than d.
It explains the long run and short run relationship among the series.
Correlation is a measure which explains the association between variables. It shows the direction of the relationship and explains how strong the relationship is. It ranges between [-1, 1].

Crosscorrelation explains the correlation among two series for different lags. Two series can be uncorrelated but cointegrated.

PART B.
Step 1. ADF-Unit roots test on residuals:
\( H_0 \) : The series has a unit root
\( H_a \) : The series has no unit root
Test statistics is: \( t = -4.85 \)
Mac Kinnon Table with 1% significance level gives:
\[ K = -3.90 + (-10.53)(461)^{-1} + (-30.03)(461)^{-2} = -3.923 \]
Mac Kinnon Table with 5% significance level gives:
\[ K = -3.35 \]
Mac Kinnon Table with 10% significance level gives:
\[ K = -3.055 \]
As \( |t| > |K| \) we reject the null hypothesis. The residuals are stationary, i.e. there exists no unit root and two series are cointegrated.

Step 2. The coefficients of residuals for three lags are all significant based on the probabilities yielding a significance level of zero.

PART C. Error correction Model
Step 1. Residual analysis of DLIP
Test statistics \( t = -6.133 \) is significant.
This shows a long run relation of the orders received on the industrial production
\( (\approx \frac{1}{-0.1957}) = 5 \text{ years} \).
Step 2; Coefficient analysis of DLIP
For the short run relationship: DLIP(-1), DLIP(-2), DLORD(-1) have significant effect on DLIP.
That is, Industrial production is significantly affected by its first and second previous observations and also the first previous observation of the orders received.

Therefore, In the long-run the industrial production is oriented on the orders received.

Step 3. Residual analysis of DORD
Test statistics \( t = 1.007 \) is not significant.
This shows a long run relation of the industrial production on the orders received is not significant.

Step 4. Coefficient analysis of DORD
The orders received is significantly influenced by DLIP(-2), DLIP(-3), DLORD(-1), DLORD(-2).
Short-run relation of orders received to its two lags previous occurrences and 2 and 3 lags previous occurrences of industrial production are significant.

QUESTION 5
PART A. The visual inspection of the graph tells us there is a structural break in 1975 as the series jump up to another level. Around 1980 and 1980, the series appear to have breaks. These may also be regarded as cyclical movement.
PART B.
Chow Breakpoint test shows the log-likelihood ratio statistics is significant which tells us a breakpoint in 1975. The hypotheses in Chow test are
\[ H_0 : \text{There exists NO structural breakpoint} \]
\[ H_a : \text{There exists structural breakpoint} \]
The test makes the comparison of restricted and unrestricted model to determine the existence of structural breaks.