Macroeconometrics

Vector Autoregressions
Lecture 5
Vector Autoregression (VAR)

Checking for the lag length

The model should represent the observed processes as precise as possible along with attaining error terms to be at minimum. Therefore, the choice of the number of variables to be included into the model is important.

If the lag length is chosen to be too short, serial correlation among error terms become significant.

A test on the two possible choice of the order

$H_0$ : the model needs p+1 lags

( the coefficients of $y_{1,t-p}, y_{2,t-p}, \ldots, y_{k,t-p}$ are all zero)

$H_a$ : the model needs p lags
Test statistic: Log Likelihood test

\[ \lambda = \frac{\text{Likelihood}(\text{restrictedmodel})}{\text{Likelihood}(\text{unrestrictedmodel})} \]

\( \square \) Chi – square

The test is performed to check if choosing the lag \( p+1 \) lags improves the power of the test or not.

Other measure for comparison is the Squared Residuals

\[ \ln\left( \frac{\hat{\mathcal{E}}'\mathcal{E}}{T} \right) \]

Compared for both models based on the statistics: Akaike Information Criterion, Schwarz information Criterion.
Granger causality test

Technique for determining whether one time series is useful in forecasting another.

\(\{X_t\}_{t=T} \) is said to Granger-cause \(\{Y_t\}_{t=T} \) if it can be shown, usually through a series of F-tests on lagged values of \(X\) (and with lagged values of \(Y\) also known), that those \(X\) values provide statistically significant information about future values of \(Y\).

The Granger test can be applied only to pairs of variables, and may produce misleading results when the true relationship involves three or more variables.
Example: Let \( \{Y_t\}_{t \in T} \) denote GDP, \( \{Y_{2t}\}_{t \in T} \) denote consumption

\[ H_0 : \text{the coefficients of } Y_{1,t-p}, Y_{2,t-p}, \ldots, Y_{k,t-p} \text{ are all zero} \]

(equiv. to \( Y_2 \) does not Granger-cause \( Y_1 \))

<table>
<thead>
<tr>
<th>Pairwise Granger Causality test</th>
<th>GDP does not Granger Cause Cons.</th>
<th>Cons. does not Granger cause GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample: 1946:1 1995:4</td>
<td>1.39156</td>
<td>7.11192</td>
</tr>
<tr>
<td>Lags: 4 Obs</td>
<td>0.23866</td>
<td>2.4E-05</td>
</tr>
</tbody>
</table>

Consumption Granger Cause on GDP.
The aim is to determine if there exist a structural change in the relationship. Fit models separately for subsample to see if there are significant differences in the estimated equations. The steps are:
1. Partition of data set data into subsamples at times having significant structural breakpoints.
2. Estimate model over whole sample and save residual sum of squares
3. Estimate model with different coefficients before and after the date $t_1$.
4. Calculate test statistics

$$F(t_1) = \frac{(T - 2k)(\mathbf{e}'\mathbf{e} - \mathbf{e}'\mathbf{e})}{(e'\mathbf{e})k}$$

where $\mathbf{e}$: residuals for unrestricted model, $\mathbf{e}$: residuals of restricted model, $T$: no. of observations, $k$: no. of parameters

5. Conclude that the model is stable if $F$ is below the critical value

<table>
<thead>
<tr>
<th>Chow Breakpoint Test</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-statistics</td>
<td>38.39</td>
</tr>
<tr>
<td>Log Likelihood ratio</td>
<td>65.75</td>
</tr>
</tbody>
</table>

Ho: No structural changes is rejected!
Variance Decomposition

Based on the forecasts, the variance of error terms is decomposed to reduce the uncertainty not only in one but in all series because of the interaction among the variables in VAR model.

\[ y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + \varepsilon_t \]

can be expressed as MA(\(\infty\))

\[ y_t = c + \varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \ldots = \Psi(B)\varepsilon_t \]

The matrix \(\psi_t\) has the interpretation

\[
\frac{\partial y_{t+l}}{\partial \varepsilon_t} = \psi_l
\]

\[
MSE[\hat{y}_{t+s|t}] = E[(y_{t+s|t} - \hat{y}_{t+s|t})(y_{t+s|t} - \hat{y}_{t+s|t})']
\]

\[
MSE[\hat{y}_{t+s|t}] = \Omega + \Psi_1 \Omega \Psi_1' + \Psi_2 \Omega \Psi_2' + \ldots + \Psi_{s-1} \Omega \Psi_{s-1}'
\]

\[
\Omega = E[\varepsilon_t \varepsilon_t']
\]