CREDIT SUISSE

Copula Opinion Pooling in Asset Allocation

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The views expressed are those of the author and do not necessarily represent the views of Credit Suisse

Outline

- Methods of Asset Allocation and Portfolio Theory
- Returns, Statistical Moments and Distributions
- The Copula Opinion Pooling (COP) Approach following Meucci
- COP- Application
- Summary and Outlook
- References
- Disclaimer

Throughout the presentation, simulation results will be shown, which are based on MATLAB codes and scripts. The codes are either written by CSAM Real Estate Strategy and Portfolio Solutions or are adjusted and rewritten codes based on the original scripts of Attilio Meucci, publicly available at www.symmys.com. The author would like to thank Attilio Meucci for helpful comments. The author is responsible for possibly remaining errors.

Methods of Asset Allocation and Portfolio Theory

History (partial)

- Markowitz 1952/1959: Portfolio Selection
- Markowitz/Sharpe 1959/1963: Single-Index-Model, Market Model
- Sharpe/Lintner/Mossin 1964-1966: Capital Asset Pricing Model (CAPM)
- Zellner/Chetty 1965: Maximization of Expected Utility
- Cohen/Pogue 1967: Multi-Index-Model
- Ross 1976/1977: Arbitrage Pricing Theory (APT)
- Black and Litterman 1990: BL-Optimization
- Numerous modern models, for example Robust Optimization Methods, Mean-Expected-Tail Loss, Higher-Moment CAPM, Copula-based methods, Value at Risk Minimization with heavy tails……

Methods of Asset Allocation and Portfolio Theory
Black-Litterman, (Robust) Bayesian Methods

- Black-Litterman: An equilibrium theory based method, a market distribution (the prior) is twisted by the views of an investor. Experience is pooled with estimations of the market. However, an underlying assumption is the normal distribution of the returns (can be circumvented). The views are expressed in the standard „alpha plus normal noise“ form.

- (Robust) Bayesian Allocations (see Meucci 2006c): The procedure shrinks a market sample towards the investor‘s prior. Experience is pooled with estimations of the market. Both the sample and the prior are expressed as parameters concerning location and dependence (mean, covariance). However, the returns are assumed to be normal distributed.

Returns, Statistical Moments and Distributions

Returns

- Linear, „simple“ Returns:

$$R_{t,k} = \frac{P_t}{P_{t-k}} - 1 = \frac{P_t - P_{t-k}}{P_{t-k}}$$

- Linear returns are easier to interpret economically.

- Logarithmic, „continuously compounded“ Returns:

$$r_{t,k} = \ln\left(\frac{P_t}{P_{t-k}}\right)$$

- Logarithmic returns are symmetric, have favourable properties for statistical models.

- Relationship:

$$r_t = \ln(1 + R_t) \qquad R_t = e^{r_t} - 1$$

Returns, Statistical Moments and Distributions
Statistical Moments

- 1st Moment (arithmetic and geometric sample mean shown here):

$$\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t \qquad \bar{r} = \sqrt[T]{\prod_{t=1}^{T}(1+r_t)} - 1$$

- 2nd Moment (variance,standard deviation):

$$\sigma^2(r) = \sum_{t=1}^{T}(r_t - \bar{r})^2 \qquad \sigma(r) = \sqrt{\sum_{t=1}^{T}(r_t - \bar{r})^2}$$

- 3rd Moment (sample skewness)

$$\hat{s}(r) = \frac{1}{T}\left[\frac{(r_t - \bar{r})^3}{\sigma^3(r)}\right]$$

- 4th Moment (sample kurtosis)

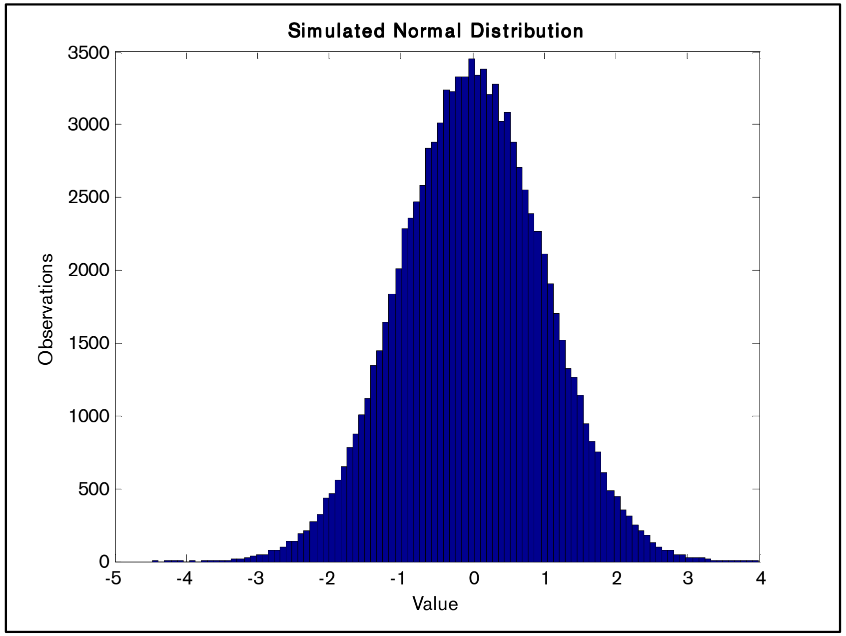
$$\hat{k}(r) = \frac{1}{T}\left[\frac{(r_t - \bar{r})^4}{\sigma^4(r)}\right]$$

- Note: geometric mean of linear returns = arithmetic mean of log returns

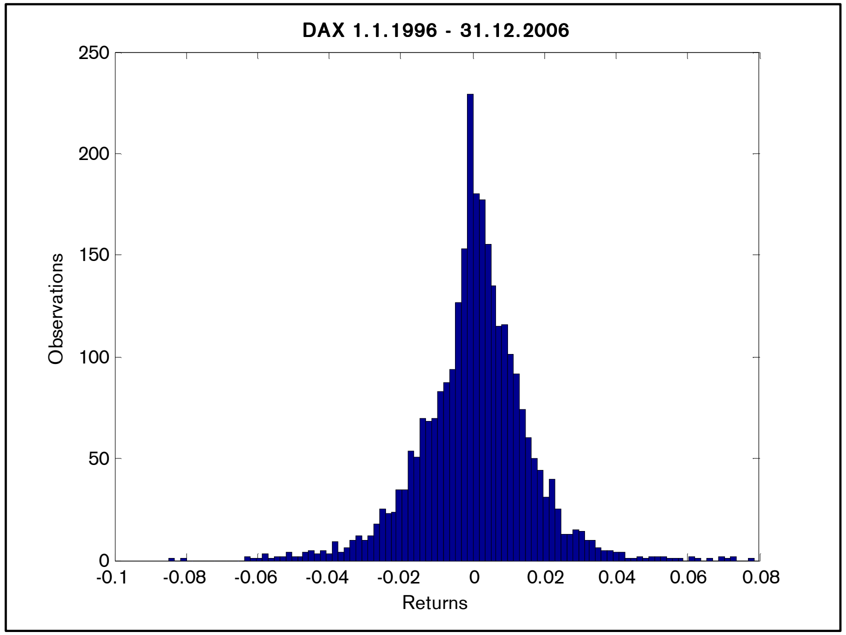
- Additional higher moments exist, but are not of relevance here.

Returns, Statistical Moments and Distributions
Distributions- univariate

- Traditional models are often mean and variance or standard deviation oriented, assume a normal distribution (skewness=0, kurtosis=3).
- Example: simulation of a normal distribution with 100.000 observations
- Skewness=0,004 and kurtosis=2,99



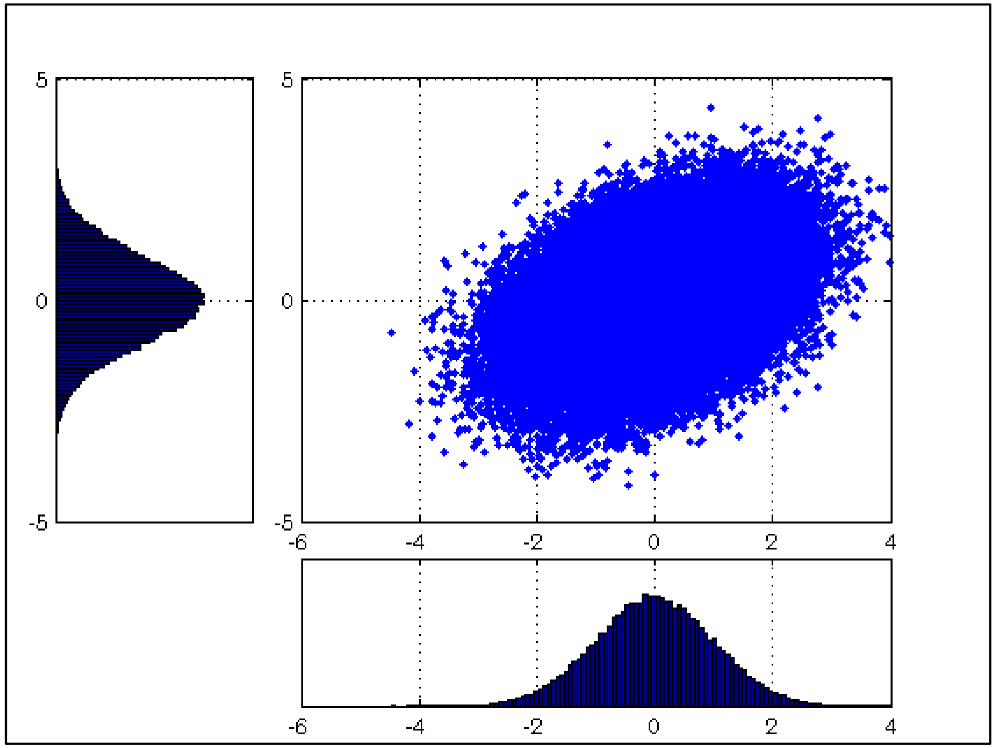
- Equity returns are often negatively skewed and/or have leptokurtotic properties (skewness<0, kurtosis>3).
- Example: DAX 30, 01/01/1996 – 12/31/2006, closing data, 2869 daily log returns
- Skewness=-0,14 and kurtosis=6,09

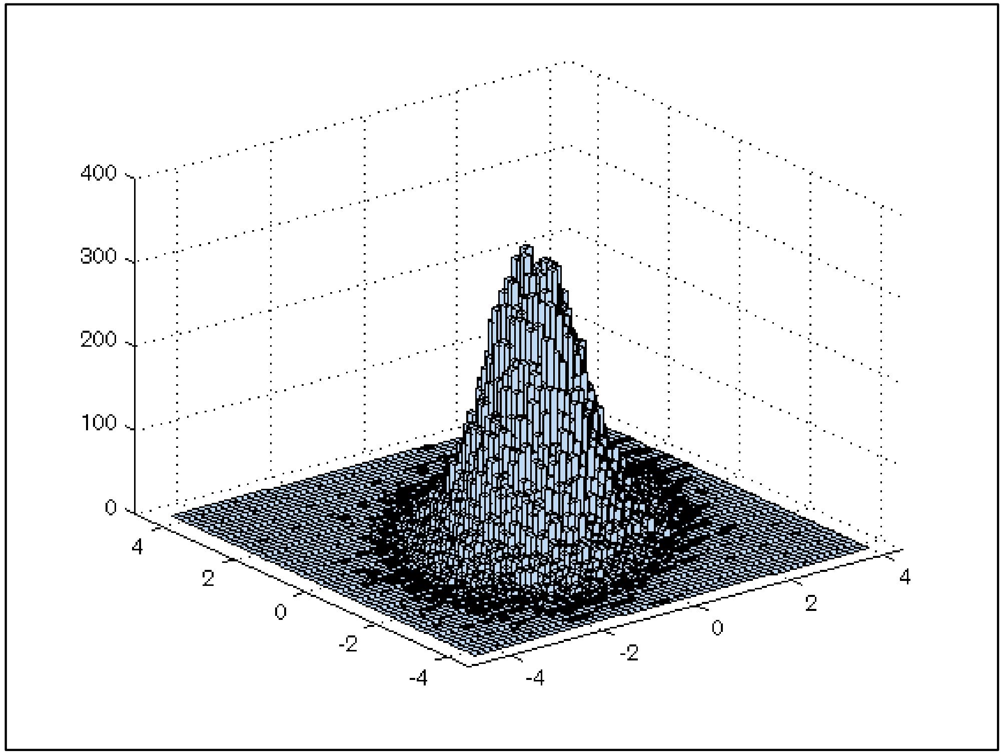


Database: Total Return Index 01/01/1996 – 12/31/2006, Source: Datastream

Returns, Statistical Moments and Distributions
Distributions- multivariate

- Example 1: Simulation of a bivariate normal distribution, 100.000 simulations
- mean=(-0,005 ; -0,003), standard deviation=(0,999 ; 0,999)
- skewness=(-0,008 ; -0,009), kurtosis=(3,015 ; 3,027)
- Correlation= 0,401 ←

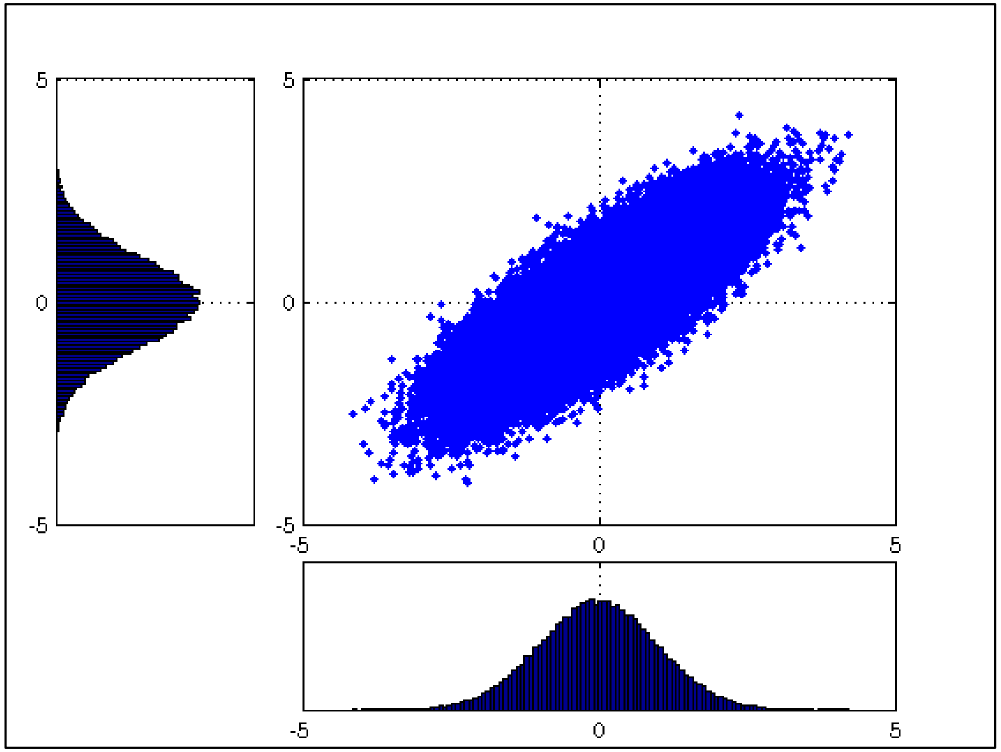


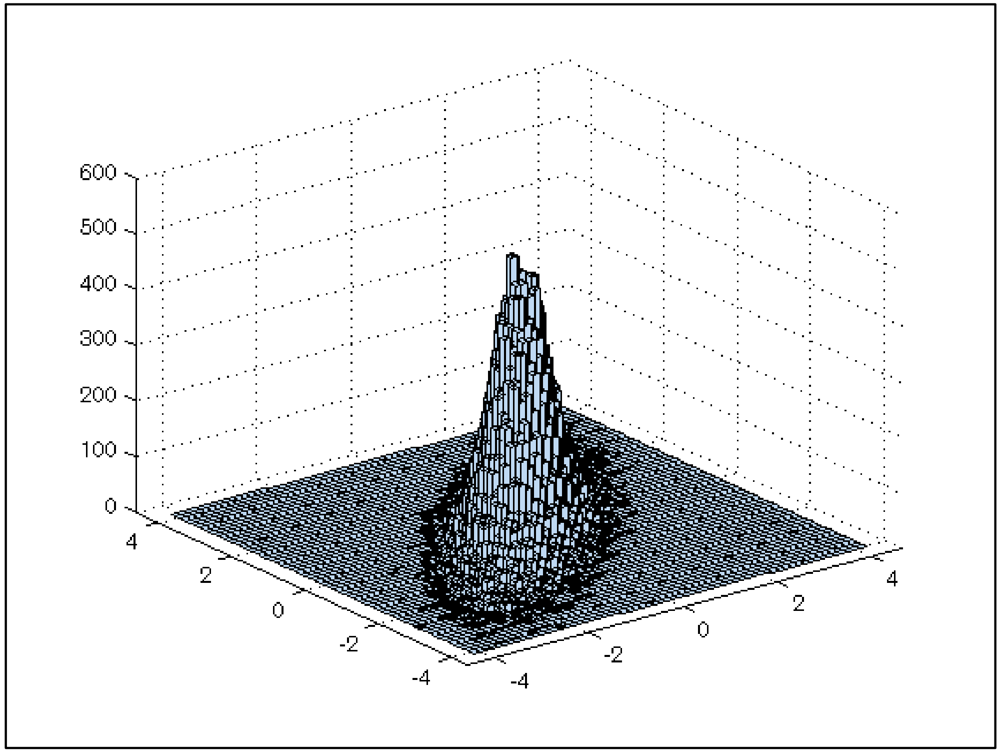


Returns, Statistical Moments and Distributions

Distributions- multivariate

- Example 2: Simulation of a bivariate normal distribution, 100.000 simulations
- mean=(-0,005 ; -0,004), standard deviation=(0,997 ; 0,999)
- skewness=(0,003 ; 0,001), kurtosis=(3,031 ; 3,012)
- Correlation= 0,801 ←





Returns, Statistical Moments and Distributions
An Introduction to Copulas

- Drawbacks of correlation: Linear correlations aren‘t sufficient for measuring interdependence among variables, for example correlation coefficients are sensitive to non-linear transformations; see Rachev, Menn and Fabozzi (2005) and Meucci (2006c).
- When the standard deviation of one variable is getting very large, the linear correlation measure may not be able to track the dependence between the variables, see Embrechts, McNeil and Straumann (2002).
- Copulas may be used for any kind of relationship between variables.
- The copula is a standardized measure of the purely joint features of a multivariate distribution.
- For an introduction to copulas see Nelsen (2006).

Returns, Statistical Moments and Distributions
An Introduction to Copulas

- Factorization of a distribution into the copula (true interdependence structure between the variables) and the marginal distribution (randomness of the variables) for each variable of a multivariate distribution.
- Transformation: the grade (uniformly distributed between 0 and 1) of a random variable is the cumulative distribution function of a one-dimensional random variable, the copula is the distribution of the grades.
- The copula of a multivariate random variable is the joint distribution of its grades.
- Origination readings: Sklar (1959, 1973)

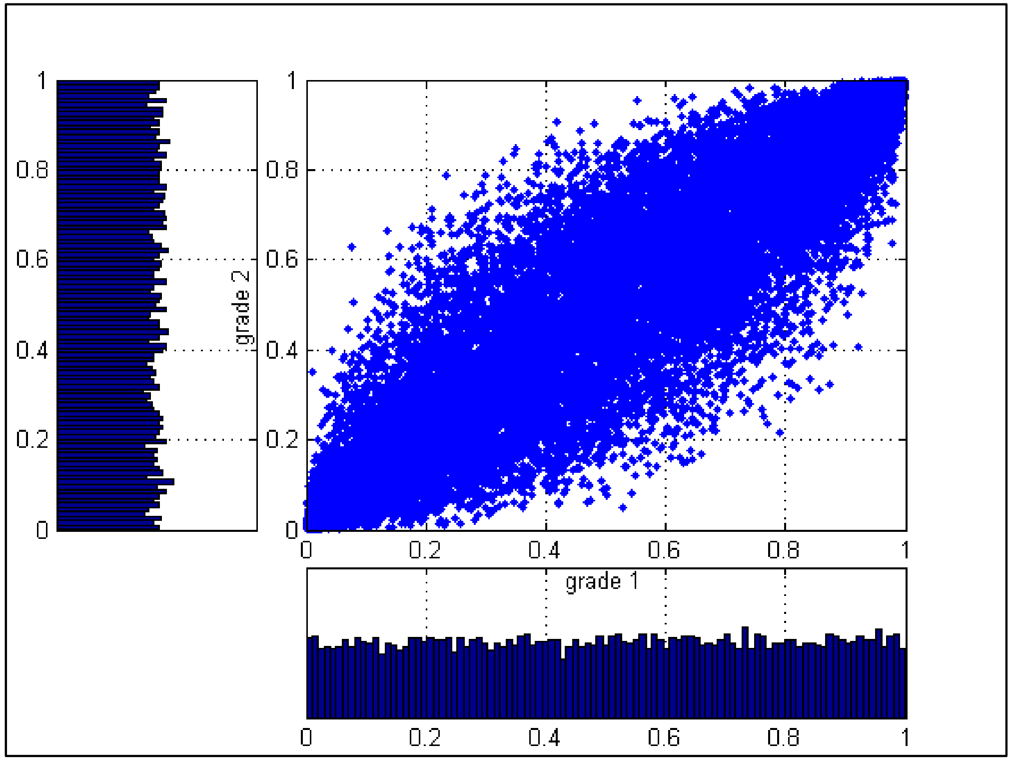
Returns, Statistical Moments and Distributions
An Introduction to Copulas

- Several possibilities concerning the shape of the copula, and therefore of the dependence structure, exist.
- Normal/ Gauss Copula: Elliptical/ radial symmetric shape, so upper and lower tail dependence is identical. However, no tail dependence is existing for the Normal Copula.
- Student/ t Copula: Elliptical/ radial symmetric shape, upper and lower tail dependence is identical, tail dependencies are determined by the degrees of freedom and the correlation.
- Elliptical copulas have no closed-form solution, numerical solution necessary.
- Clayton Copula: Archimedian class of copulas, lower tail dependence.
- Gumbel/ Logistic Copula: Archimedian class of copulas, upper tail dependence.

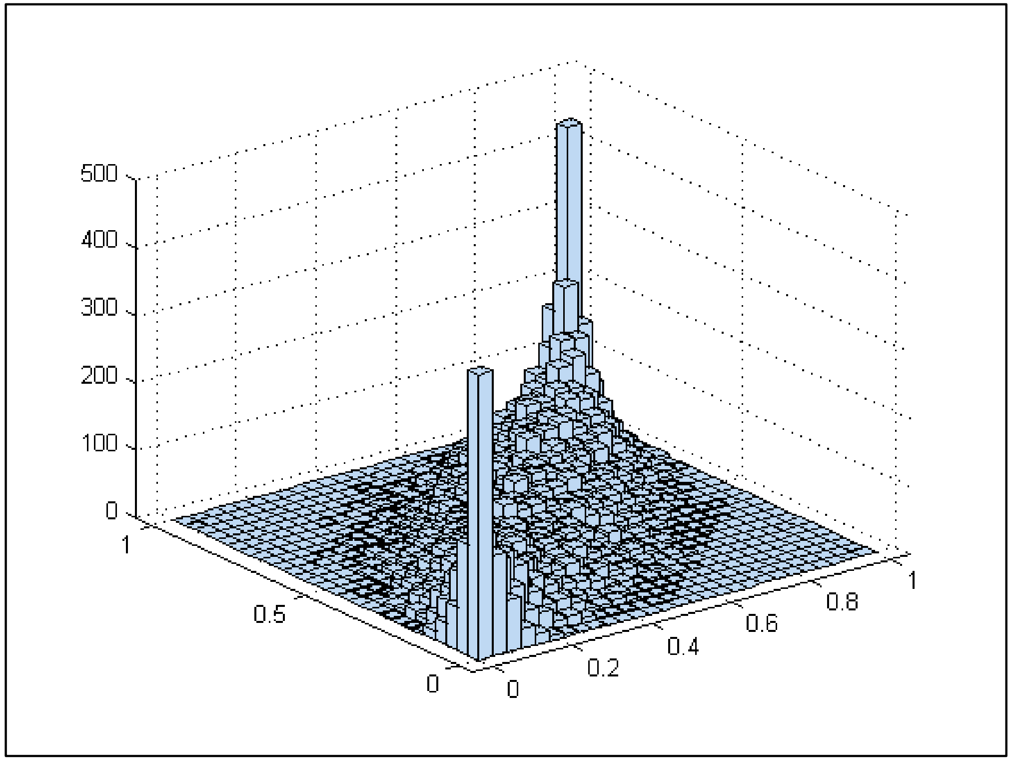
Returns, Statistical Moments and Distributions
An Introduction to Copulas

- Example: Simulation of a normal copula
- Correlation= 0.9

marginals and grades:



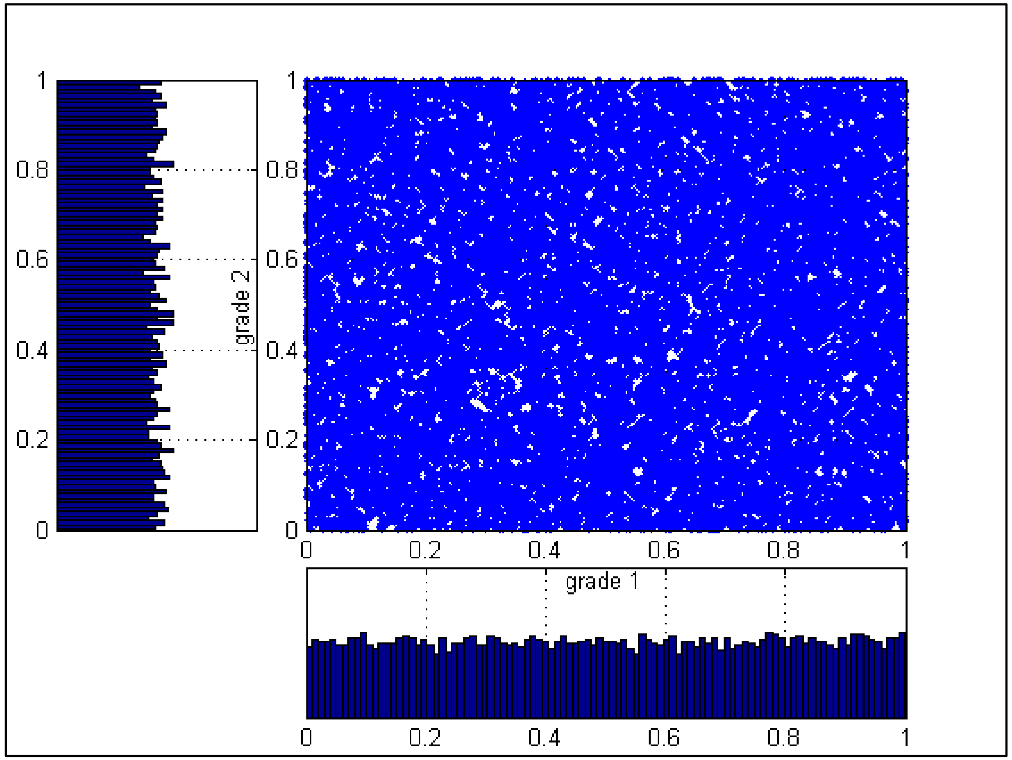
pdf of the copula:



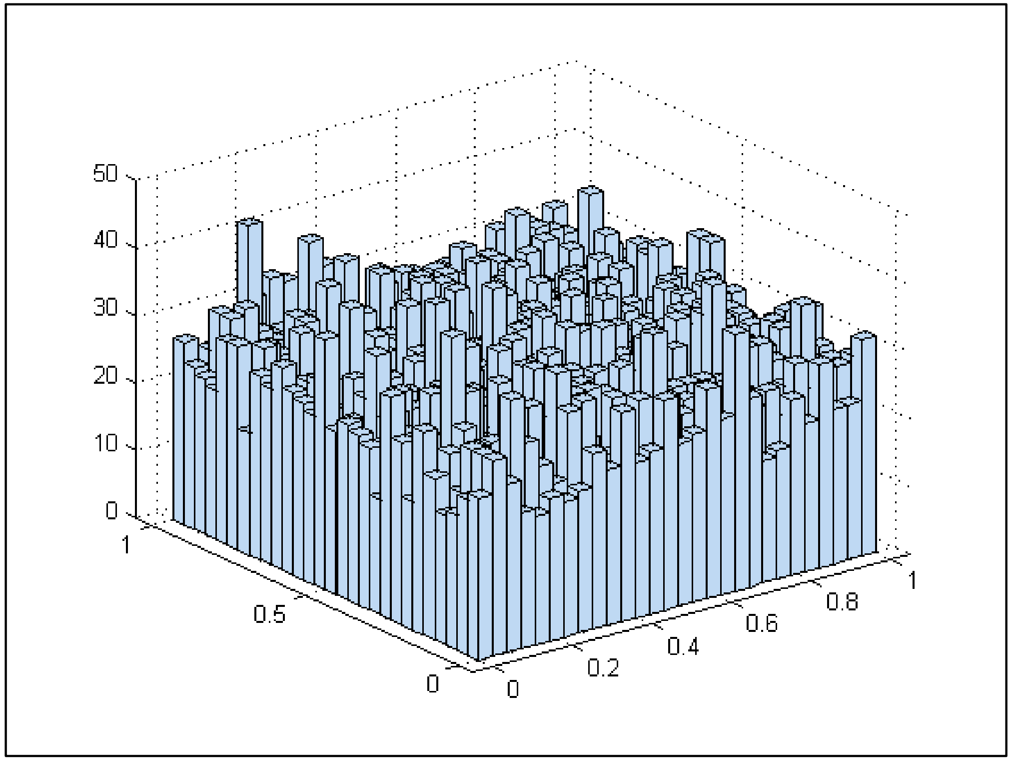
Returns, Statistical Moments and Distributions
An Introduction to Copulas

- Example: Simulation of a normal copula
- Correlation= 0

marginals and grades:



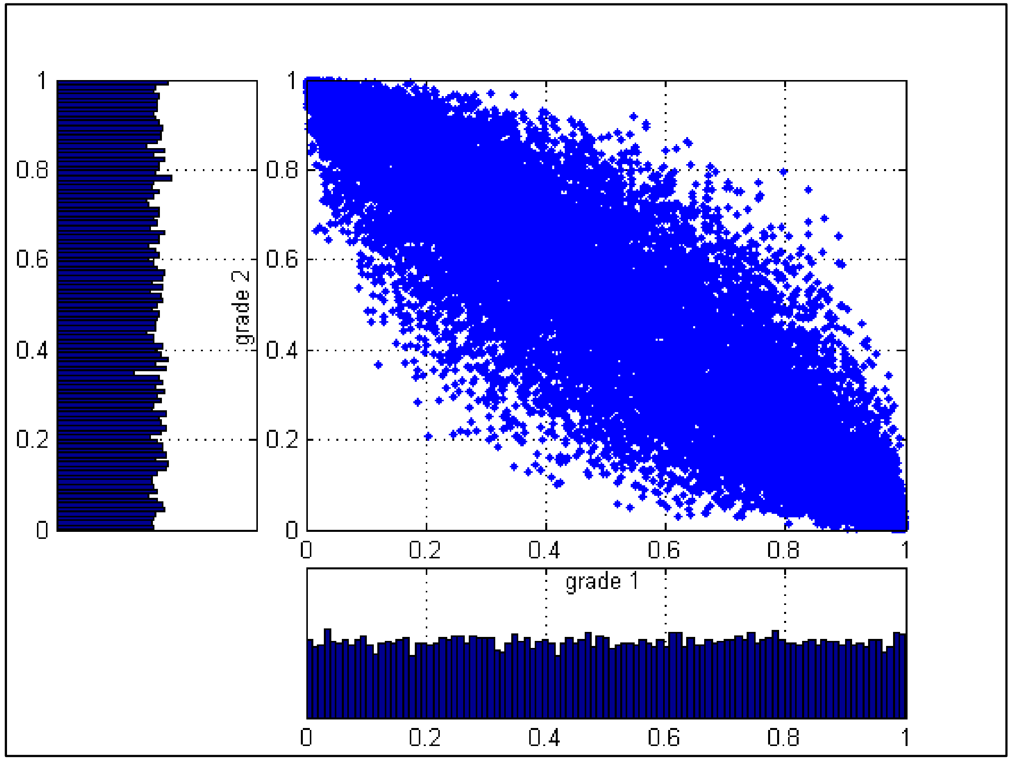
pdf of the copula:



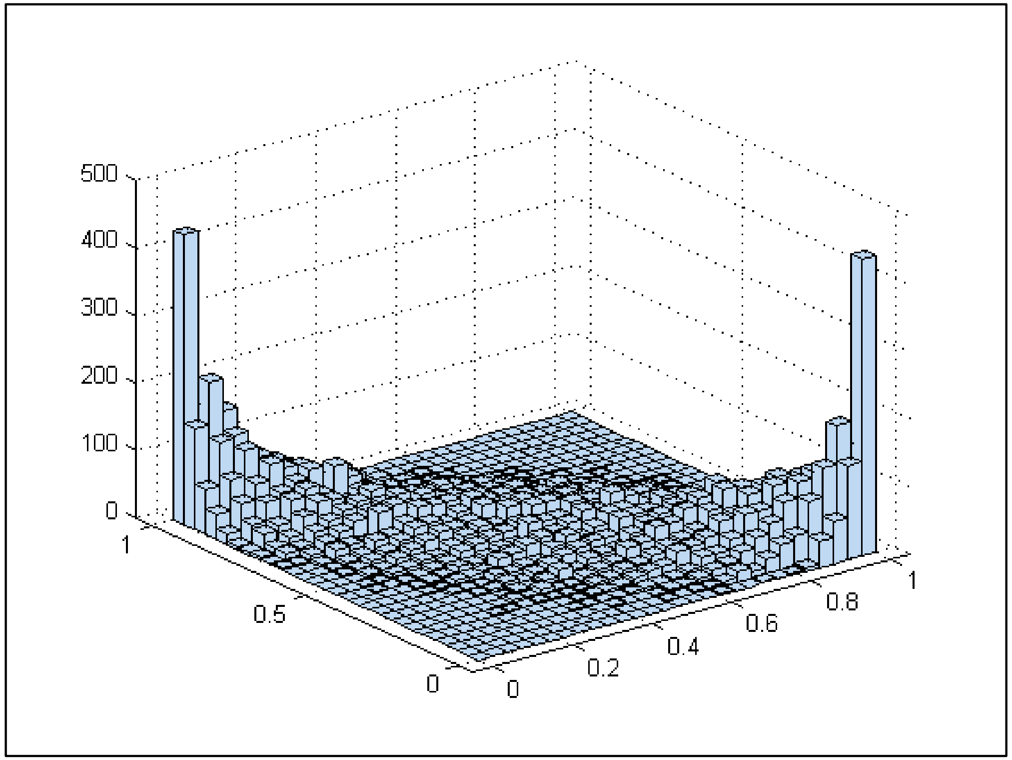
Returns, Statistical Moments and Distributions
An Introduction to Copulas

- Example: Simulation of a normal copula
- Correlation= -0.9

marginals and grades:



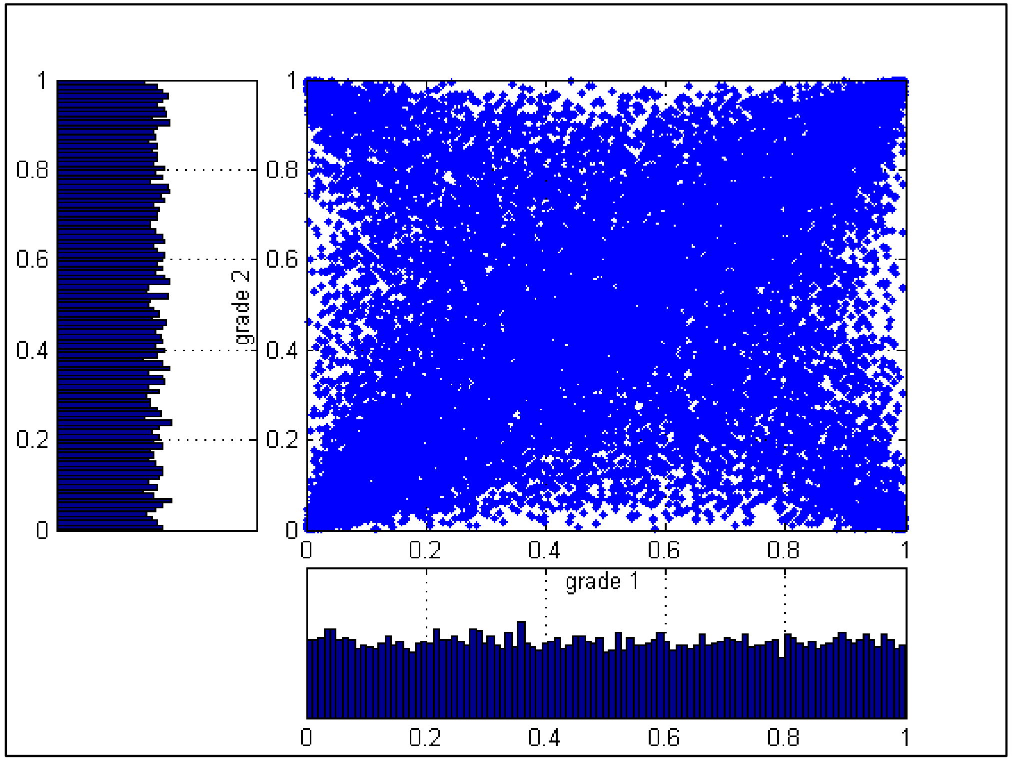
pdf of the copula:



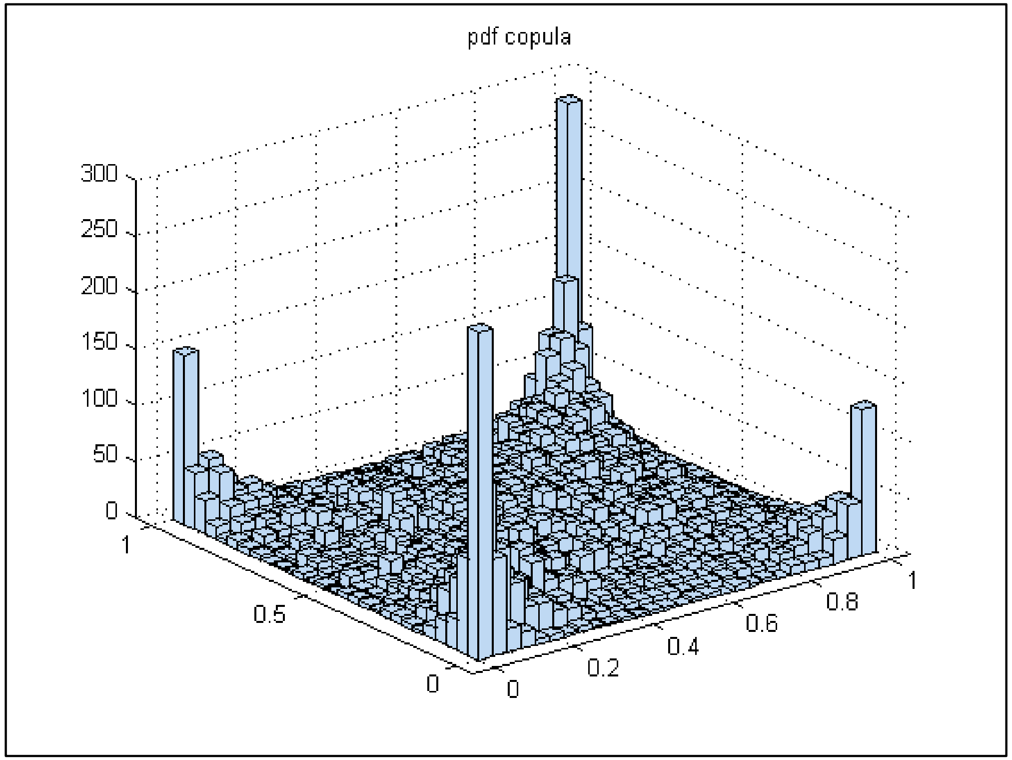
Returns, Statistical Moments and Distributions
An Introduction to Copulas

- Example: Simulation of a t copula
- Correlation= 0.3, Degrees of freedom= 1

marginals and grades:



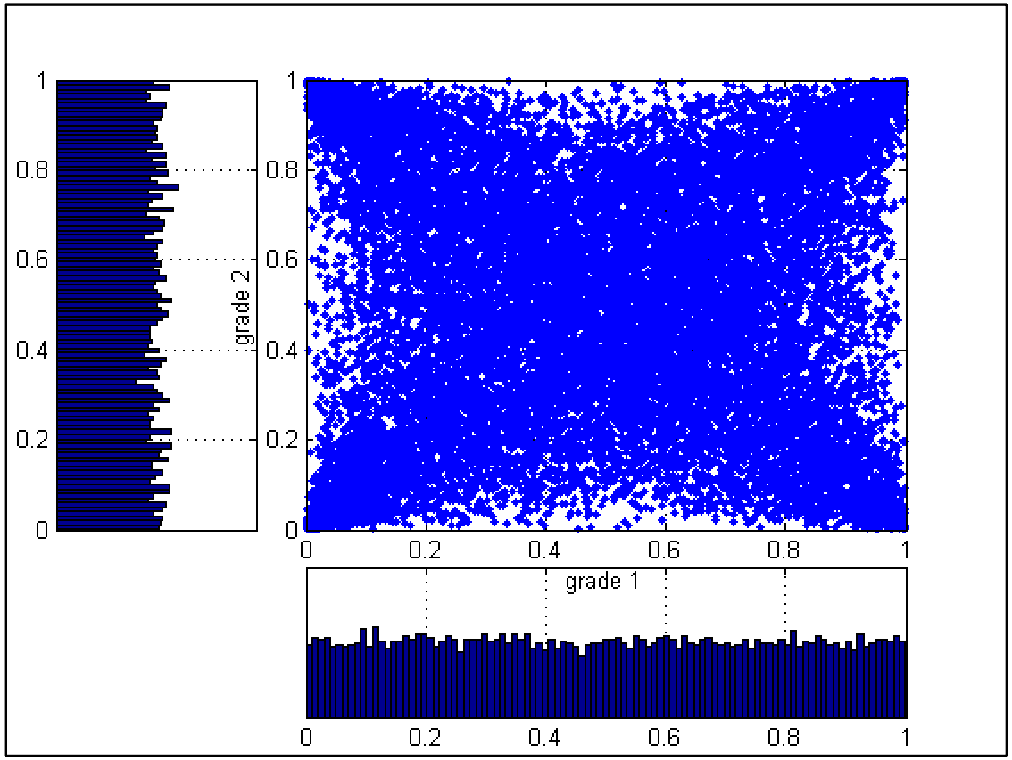
pdf of the copula:



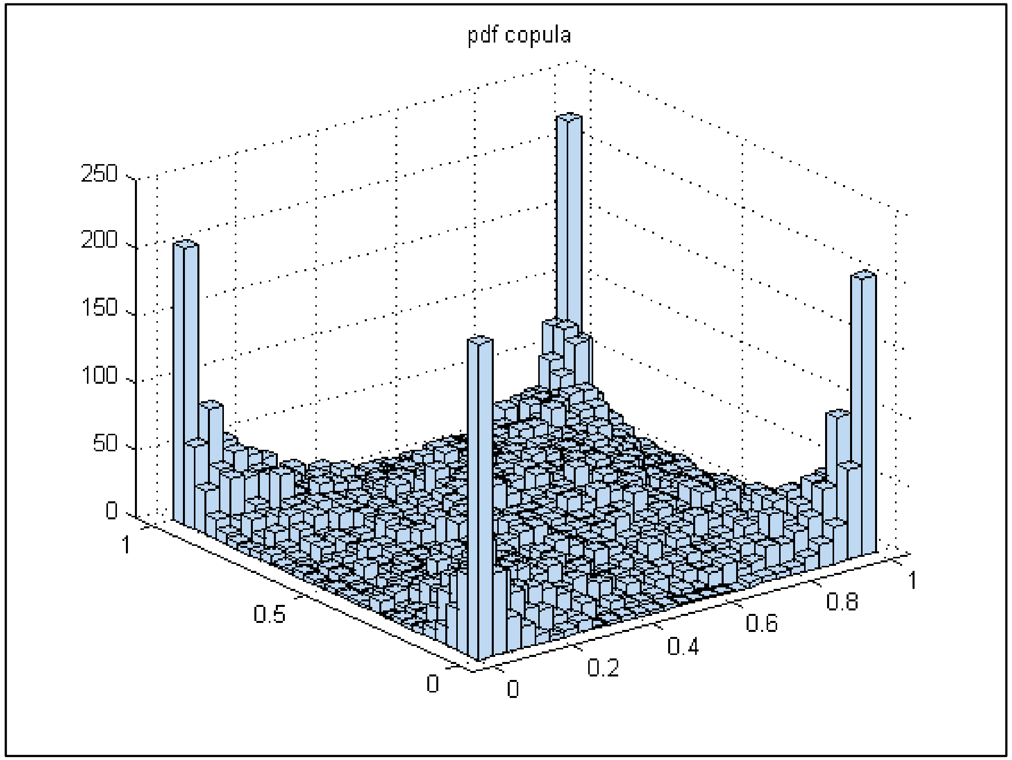
Returns, Statistical Moments and Distributions
An Introduction to Copulas

- Example: Simulation of a t copula
- Correlation= 0, Degrees of freedom= 1

marginals and grades:



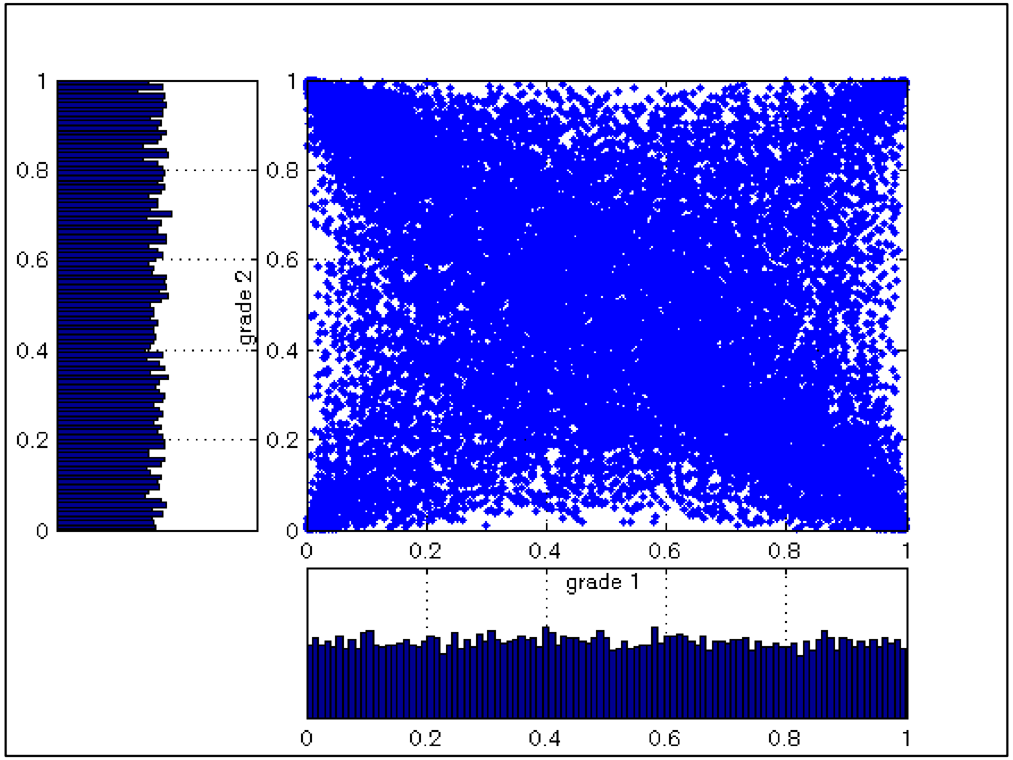
pdf of the copula:



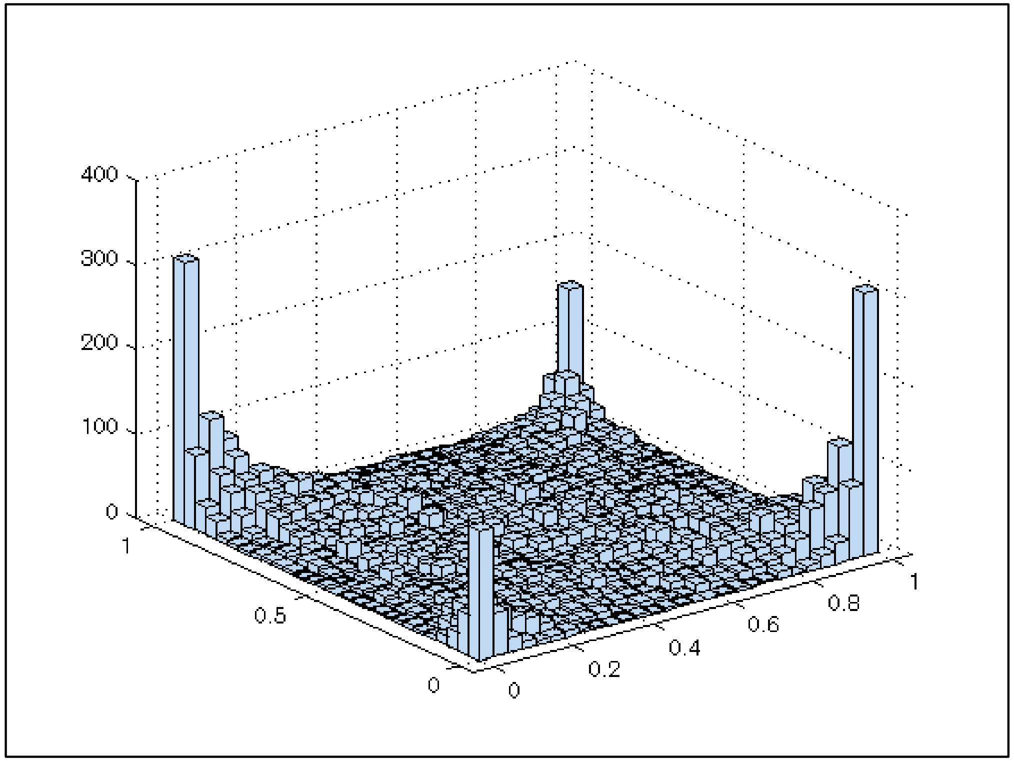
Returns, Statistical Moments and Distributions
An Introduction to Copulas

- Example: Simulation of a t copula
- Correlation= -0.3, Degrees of freedom= 1

marginals and grades:



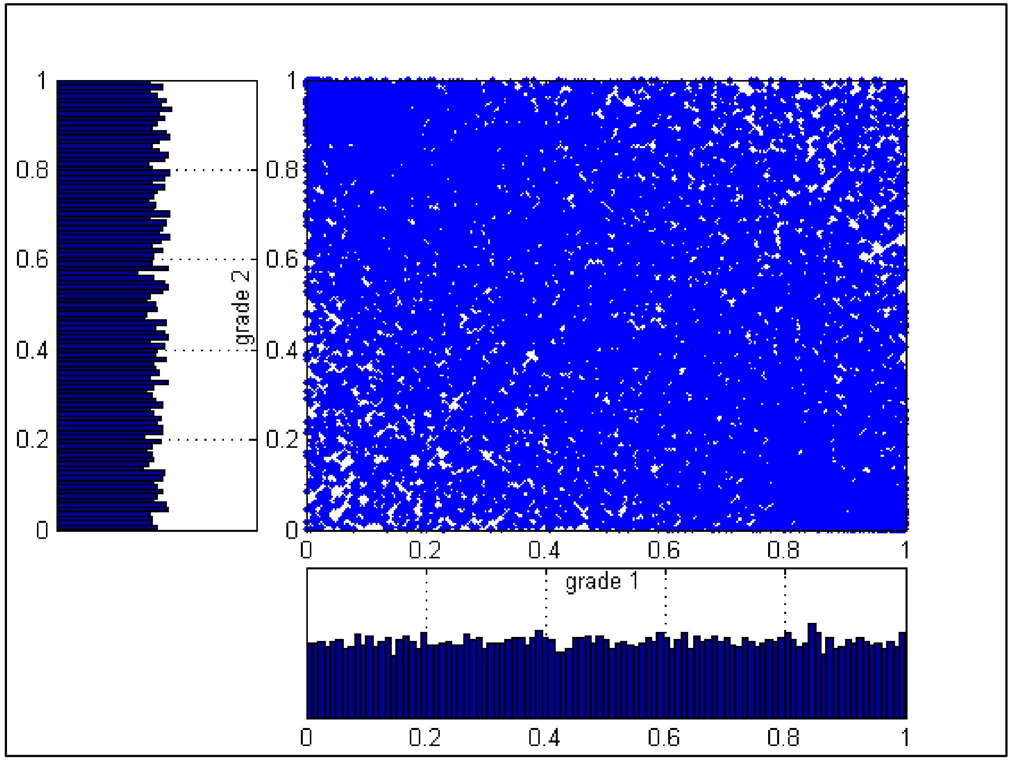
pdf of the copula:



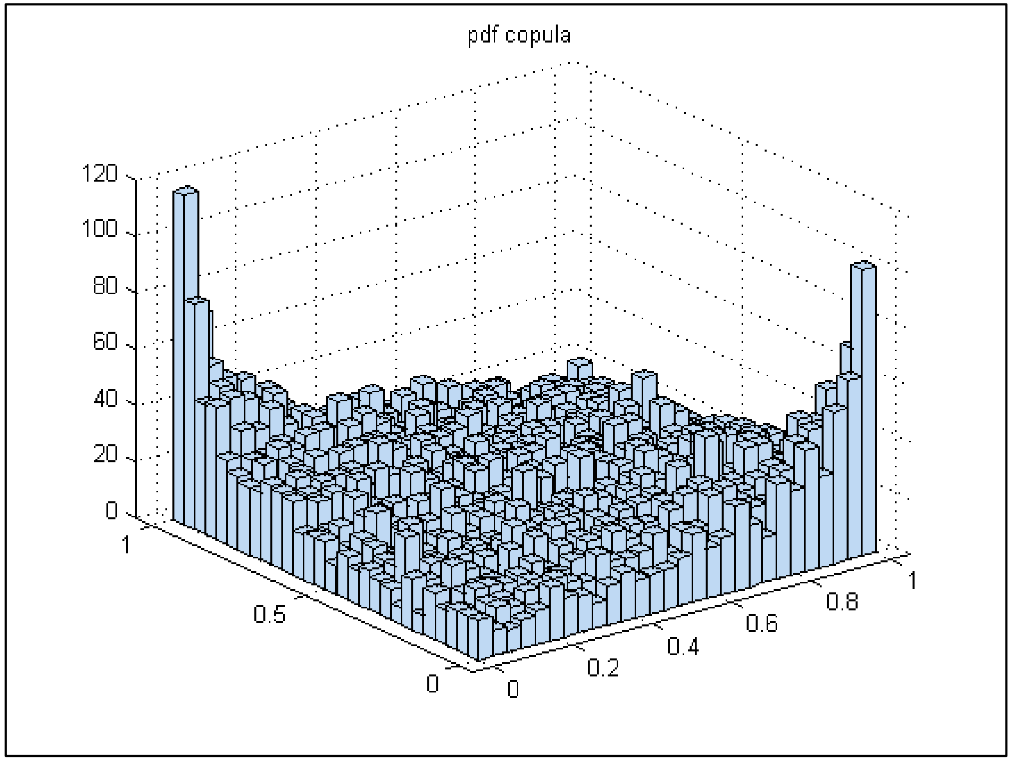
Returns, Statistical Moments and Distributions
An Introduction to Copulas

- Example: Simulation of a t copula
- Correlation= -0.3, Degrees of freedom= 10

marginals and grades:



pdf of the copula:



Returns, Statistical Moments and Distributions

An Introduction to Copulas

- In the approach subject to this presentation , a t copula is used to model the co-dependence between the variables.
- The t copula has identical upper and lower tail dependence.
- The upper (lower) tail dependence is increasing in the correlation between the variables, and decreasing in the degrees of freedom.
- For a large amount of the degrees of freedom, both tail dependencies approach zero.

The Copula Opinion Pooling (COP) approach following Meucci (2006a, 2006b)

Introduction

- Shortcomings of the classical models have to be overcome, an approach which is not dependent on assumptions about the marginal distributions of the underlying assets is sought.
- A reliable estimation of the dependencies among assets is crucial in asset allocation, calling for the use of measures beyond covariance matrices and correlations.
- The Copula Opinion Pooling (COP) approach of Meucci (2006a, 2006b) makes the modelling of dependencies by using copulas possible.
- By simulating market scenarios, the approach is free from distributional assumptions concerning the variables used.

The Copula Opinion Pooling (COP) approach following Meucci (2006a, 2006b)

Model Properties

- The investor may express his expectations/opinions.
- Opinions are expressed on market realizations, rather than on the parameters of the market.
- COP does not assume a normal distribution of the returns, the market realizations are simulated using Monte Carlo methods, these are dimensionally unbounded.
- COP contains bayesian elements, properties of the Black-Litterman Approach
- The COP approach aims at building a posterior market distribution out of a sample of market data and the opinions of the investor.
- COP outputs can serve as inputs to allocation decisions, option pricing, risk management….

The Copula Opinion Pooling (COP) approach following Meucci (2006a, 2006b)

Model Properties

- Necessary Inputs:

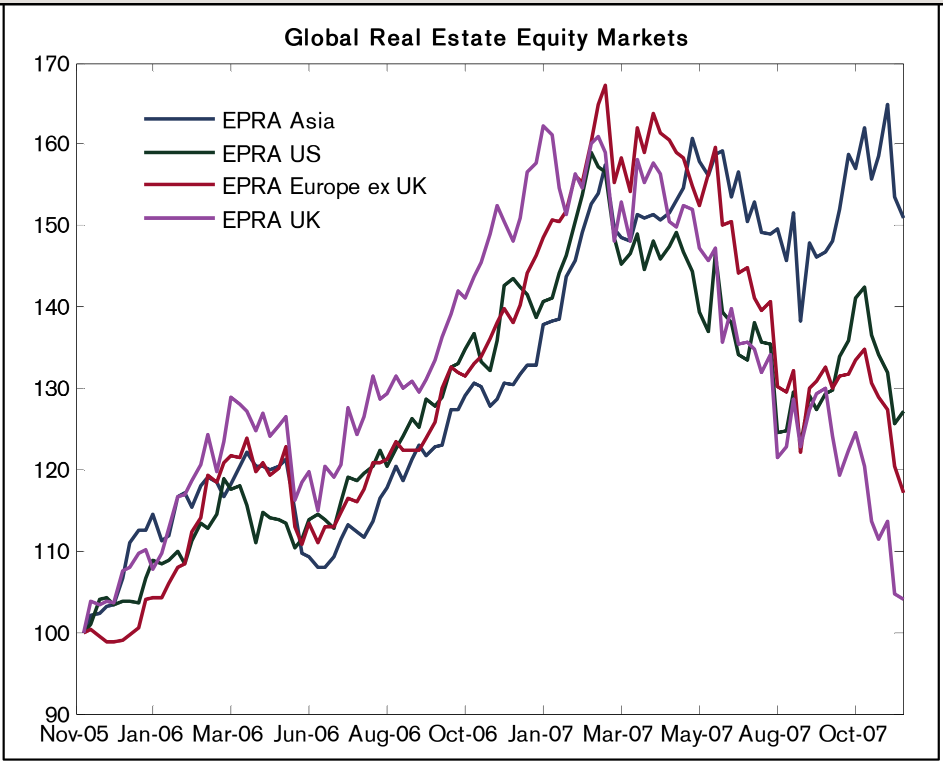
1. The „market prior“, derived from real data.
2. The investor‘s „opinions“, „views“.

- Step 1: The market prior is brought into the view matrix.
- Step 2: The prior Monte Carlo view scenarios are sorted, generating the prior cumulative distribution functions (cdf) as well as the prior copulas.
- Step 3: The marginal posterior cdf of the views, the prior and the posterior are calculated.
- Step 4: The joint posterior distributions of the views are calculated.
- Step 5. The joint posterior market realisations are calculated.

COP- Application
The Data

- Real Estate equity market indices from 11/10/2005 to 11/15/2007, weekly returns:

	EPRA Asia	EPRA US	EPRA Eur ex UK	EPRA UK
Mean	0,42%	0,26%	0,18%	0,08%
Standard Dev.	2,40%	2,42%	2,54%	3,05%
Skewness	-0,84	-0,43	-1,09	-0,77
Kurtosis	5,21	3,85	5,10	3,71

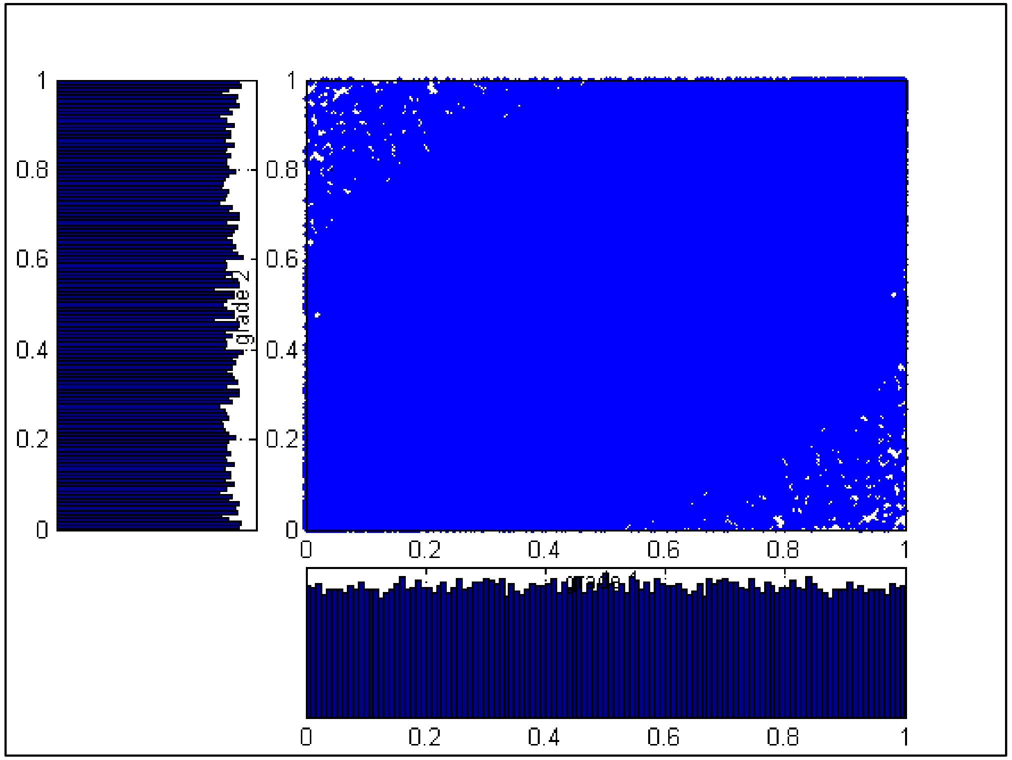


Database: Total Return Indices, 11/10/2005 – 11/15/2007, Source: Datastream

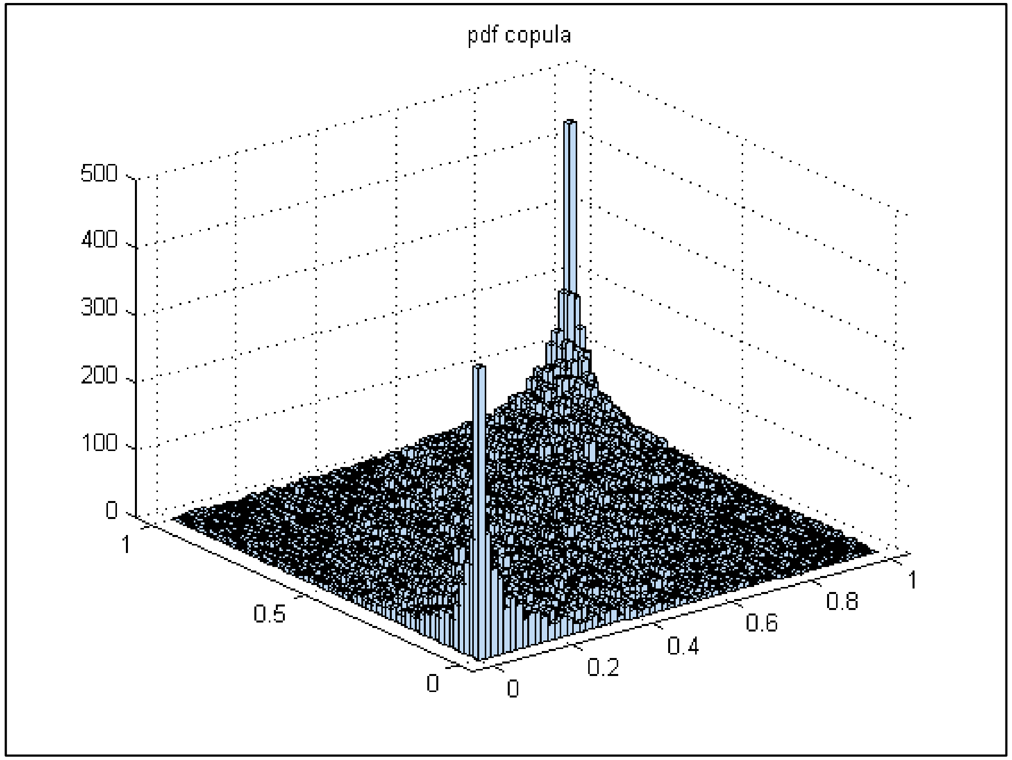
COP- Application
The Data

- Copula of EPRA Europe ex UK and EPRA United States:

marginals and grades:



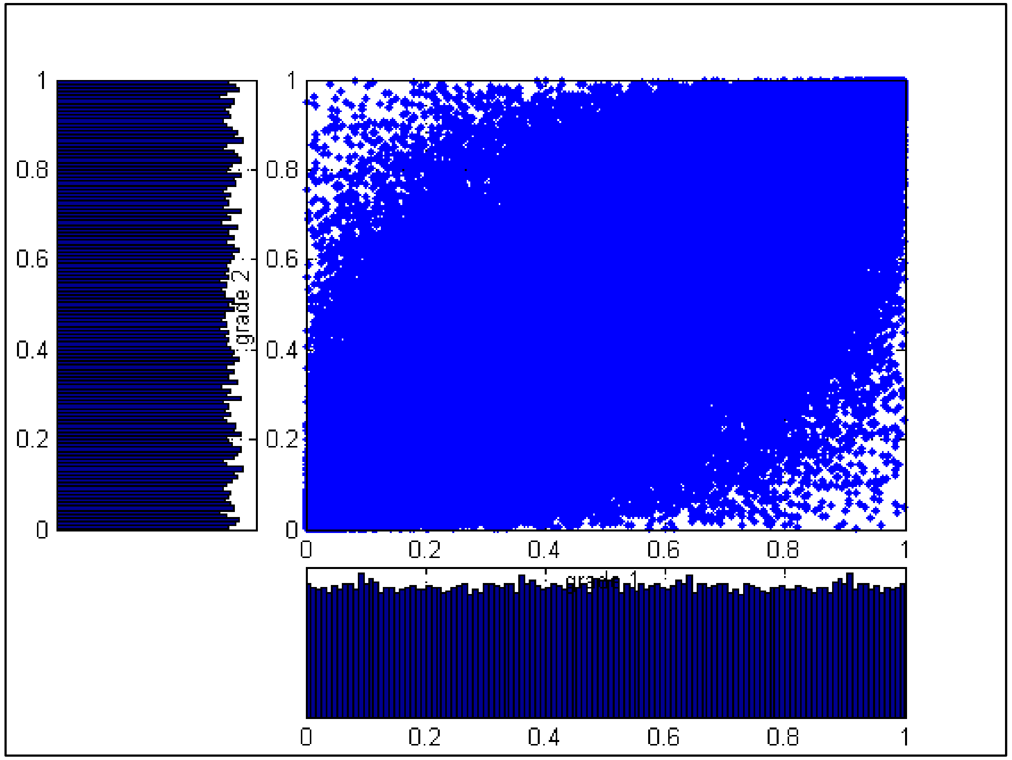
pdf of the copula:



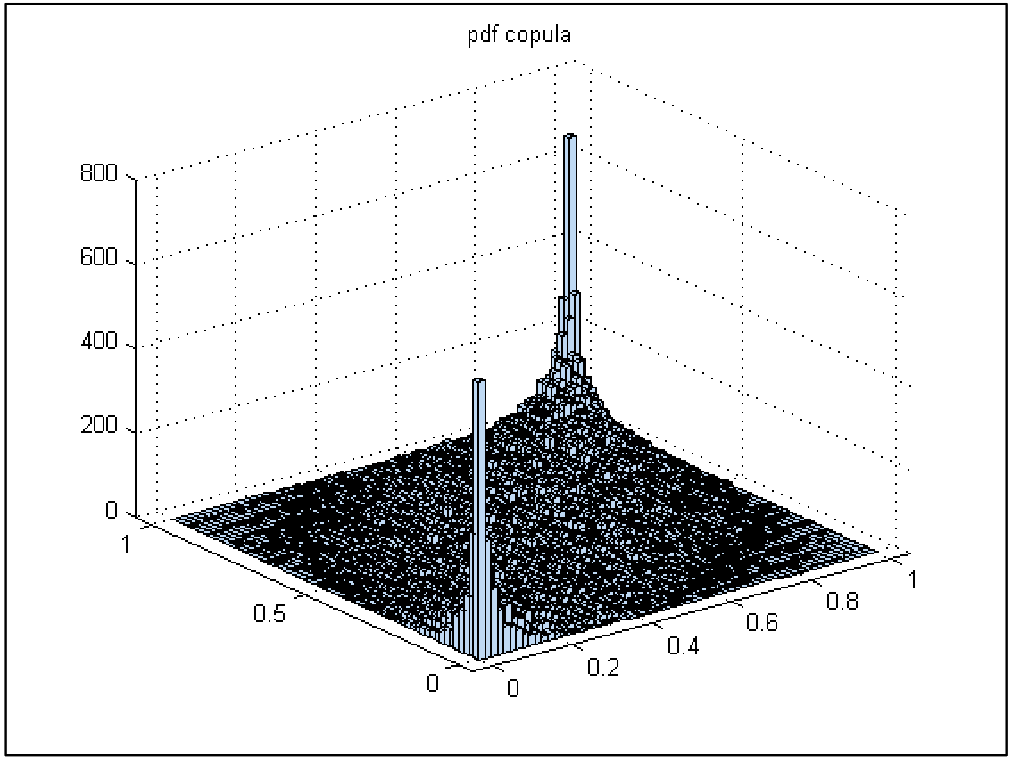
COP- Application
The Data

- Copula of EPRA Europe ex UK and EPRA United Kingdom:

marginals and grades:



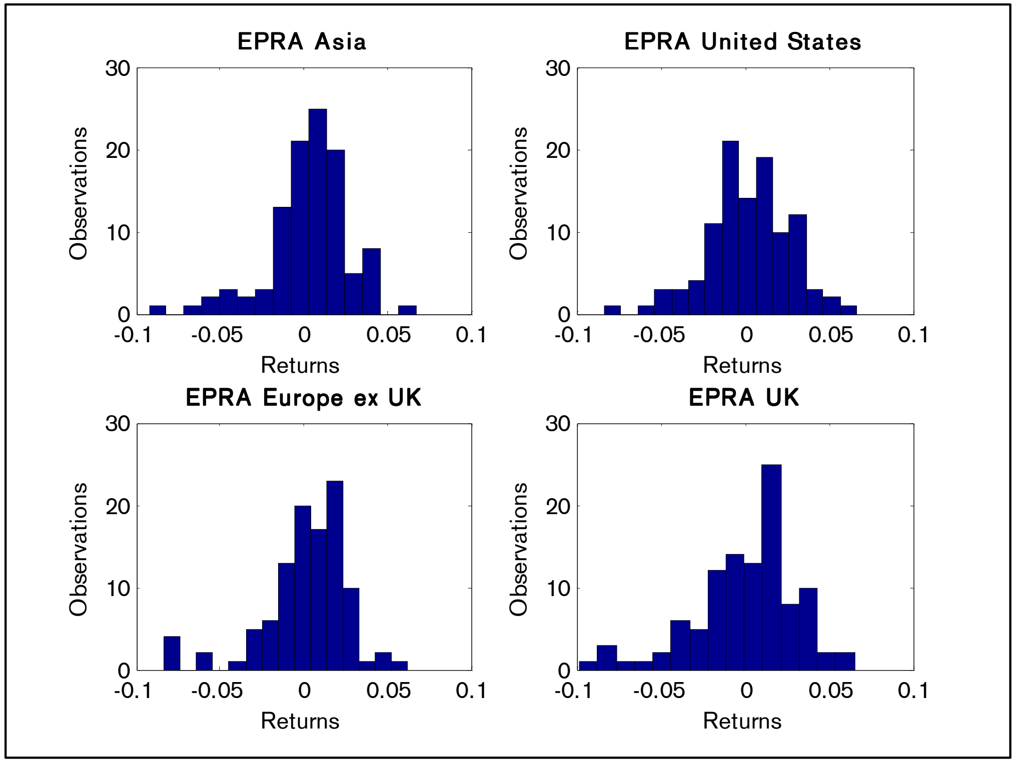
pdf of the copula:



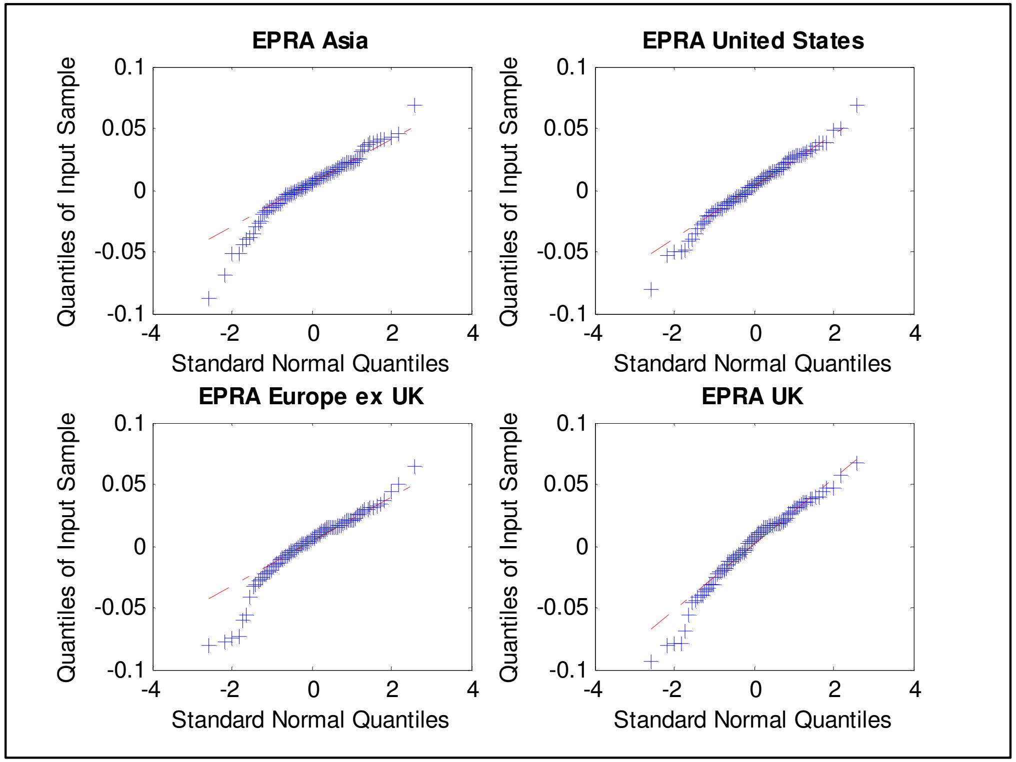
COP- Application
The Data

- Histograms of the return series of the four indices and qq-plots of return data vs. standard normal distribution show the inappropriateness of a normal assumption.

Histogram of returns of EPRA markets:



qq-plots EPRA returns against st.- normal:

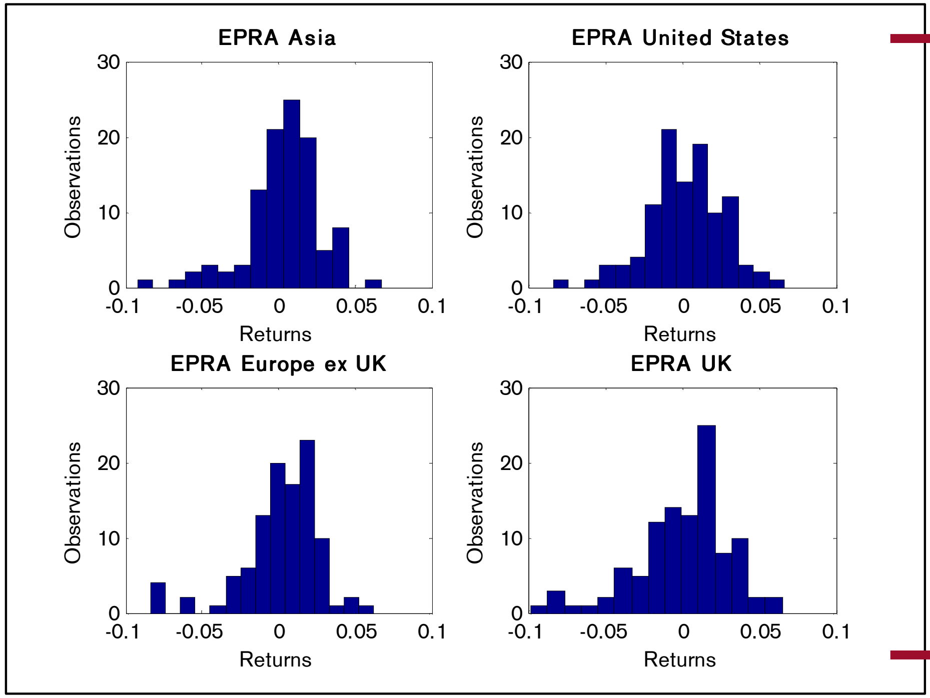


COP- Application

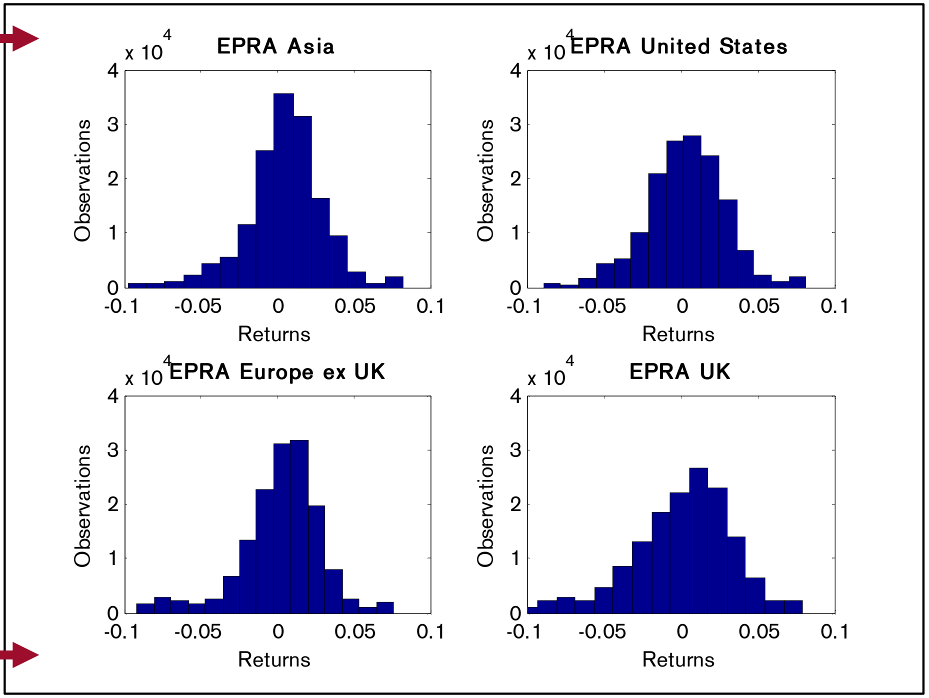
The Market Prior

- From the data, the prior market distribution is estimated.
- Marginals as well as the copulas are calculated, the market prior is yield by Monte Carlo simulations (here: 150.000 simulations)

Histogram of returns of EPRA markets:



Histogram of market prior:

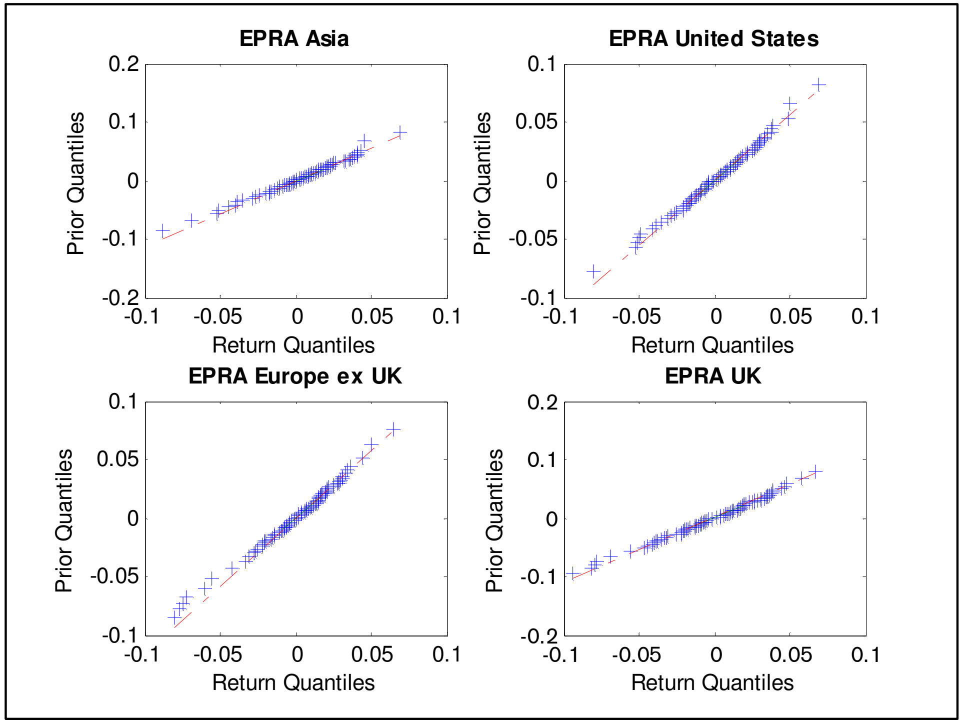


COP- Application

The Market Prior

- From the data, the prior market distribution is estimated.
- According to the qq-plots, the return sample and the simulated prior seem to be of the same distribution.

qq-plot of EPRA returns versus market prior:



COP- Application

The Market Prior

- Compare the market sample return data with the simulated prior market scenarios:

Descriptive statistics of return data of EPRA markets:

	EPRA Asia	EPRA US	EPRA Eur ex UK	EPRA UK
Mean	0,42%	0,26%	0,18%	0,08%
Standard Dev.	2,40%	2,42%	2,54%	3,05%
Skewness	-0,84	-0,43	-1,09	-0,77
Kurtosis	5,21	3,85	5,10	3,71

Descriptive statistics of market prior:

	EPRA Asia	EPRA US	EPRA Eur ex UK	EPRA UK
Mean	0,50%	0,34%	0,25%	0,16%
Standard Dev.	2,57%	2,58%	2,70%	3,23%
Skewness	-0,46	-0,14	-0,75	-0,55
Kurtosis	4,80	3,83	4,72	3,56

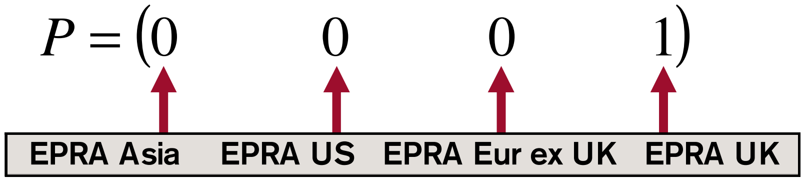
COP- Application

The Investor‘s Views

- Investor may have K<N views on the market for N assets, a „pick matrix“ is set up. The views are assumed to be uninformative (uniformly distributed between the view bounds) in opposition to the „alpha plus noise“ expressions in Black-Litterman.

- Example: Investor is bullish for UK, assumes a weekly gain of flat to 5% with a confidence level of 50%.

pick matrix:



$$P = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$$

bounds:

$$a\_b = \begin{pmatrix} 0 & 0{,}05 \end{pmatrix}$$

confidence:

$$c = \begin{pmatrix} 0{,}5 \end{pmatrix}$$

COP- Application

Step 1

- P is augmented with a matrix that is orthogonal to the views and to the market, the resulting matrix P-bar shows the view adjusted market coordinates:

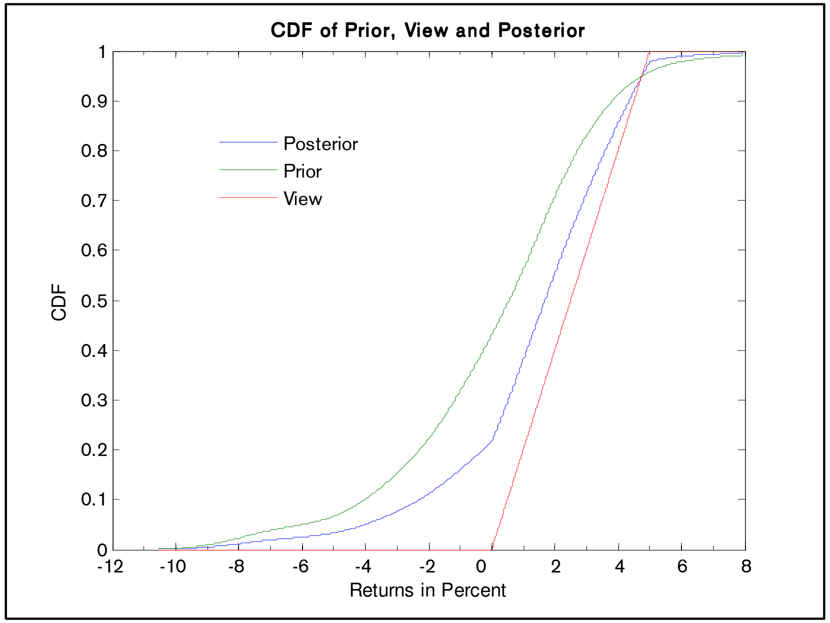
$$\overline{P} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

pick matrix

- Transposed P-bar matrix is multiplied with the market prior, the result is a 150000x4 matrix representing joint realisations of the market in the views coordinates (market-implied distribution of the views).

COP- Application
Step 2 and 3

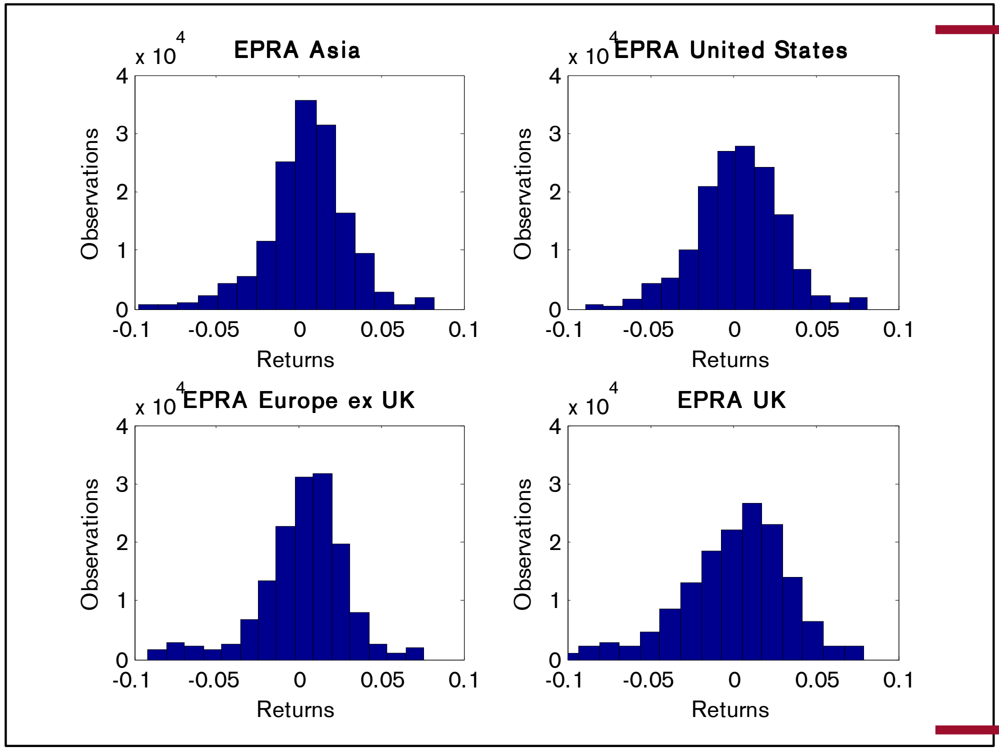
- The Monte Carlo view scenarios are sorted, generating the prior cumulative distribution functions (cdf) as well as the prior copulas.
- The marginal posterior cdf of the prior, the views and the posterior are calculated. The posterior marginal of each view represents the weighted average of the marginal market prior and the subjective views.



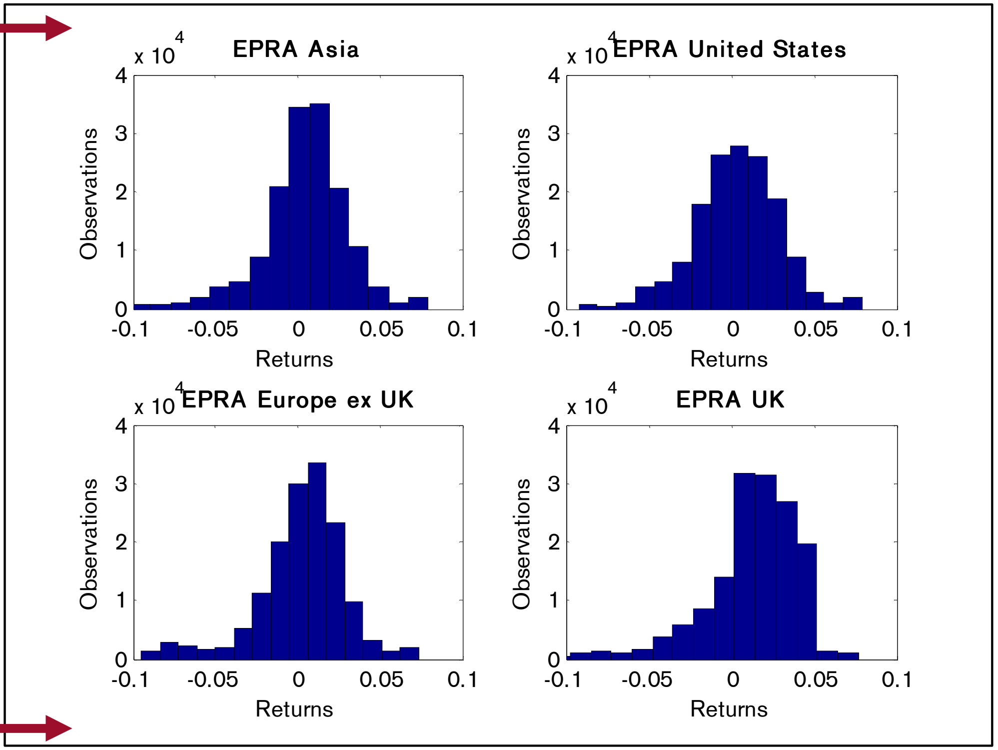
COP- Application
Step 4 and 5

- The joint posterior distributions of the views are calculated.
- The joint posterior market realisations are calculated.

Histogram of market prior:



Histogram of market posterior:



COP- Application
Assessment / Comparison

- Compare the market sample return data with the simulated prior and the posterior:

Descriptive statistics of return data of EPRA markets:

	EPRA Asia	EPRA US	EPRA Eur ex UK	EPRA UK
Mean	0,42%	0,26%	0,18%	0,08%
Standard Dev.	2,40%	2,42%	2,54%	3,05%
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Kurtosis	5,21	3,85	5,10	3,71

Descriptive statistics of market prior:

	EPRA Asia	EPRA US	EPRA Eur ex UK	EPRA UK
Mean	0,50%	0,34%	0,25%	0,16%
Standard Dev.	2,57%	2,58%	2,70%	3,23%
Skewness	-0,46	-0,14	-0,75	-0,55
Kurtosis	4,80	3,83	4,72	3,56

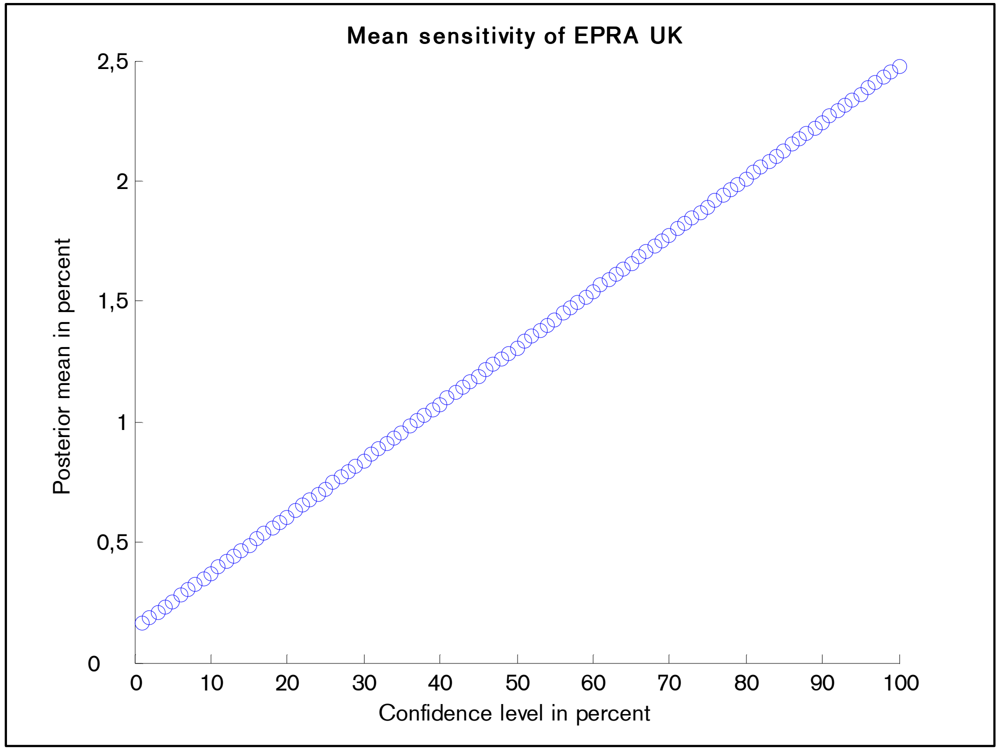
Descriptive statistics of market posterior:

	EPRA Asia	EPRA US	EPRA Eur ex UK	EPRA UK
Mean	0,50%	0,34%	0,25%	1,33%
Standard Dev.	2,57%	2,58%	2,70%	2,76%
Skewness	-0,46	-0,14	-0,75	-1,14
Kurtosis	4,80	3,83	4,72	5,06

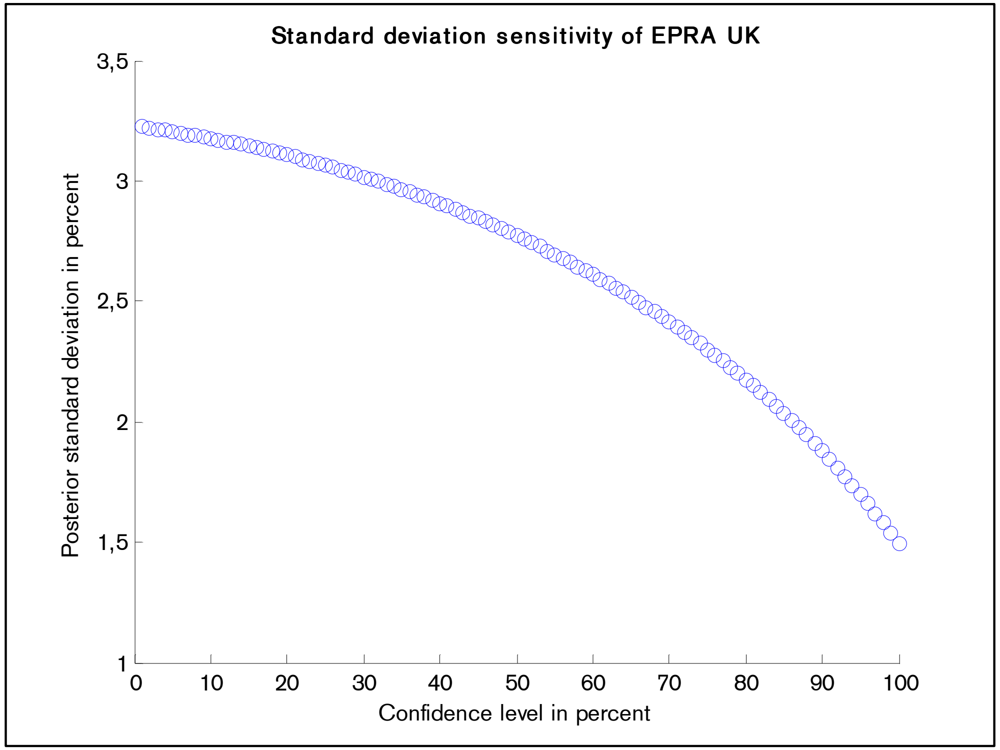
COP- Application
Assessment / Comparison

- Sensitivity of the mean and the standard deviation of the posterior weekly return of the EPRA UK for different levels of confidence in our example.

Mean sensitivity:



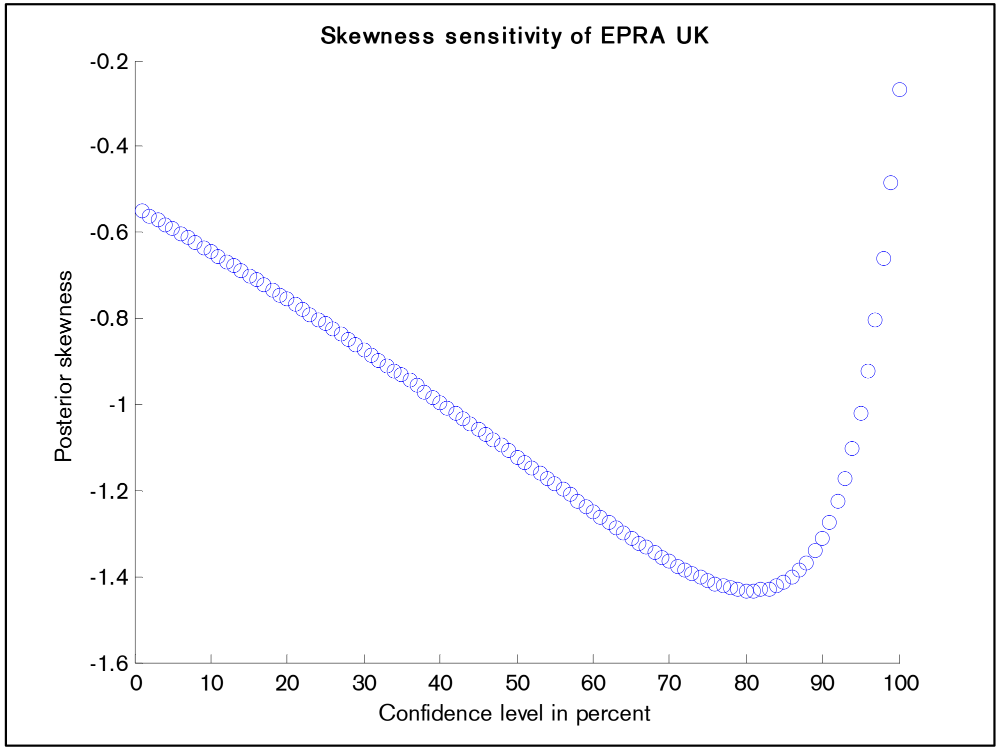
Standard deviation sensitivity:



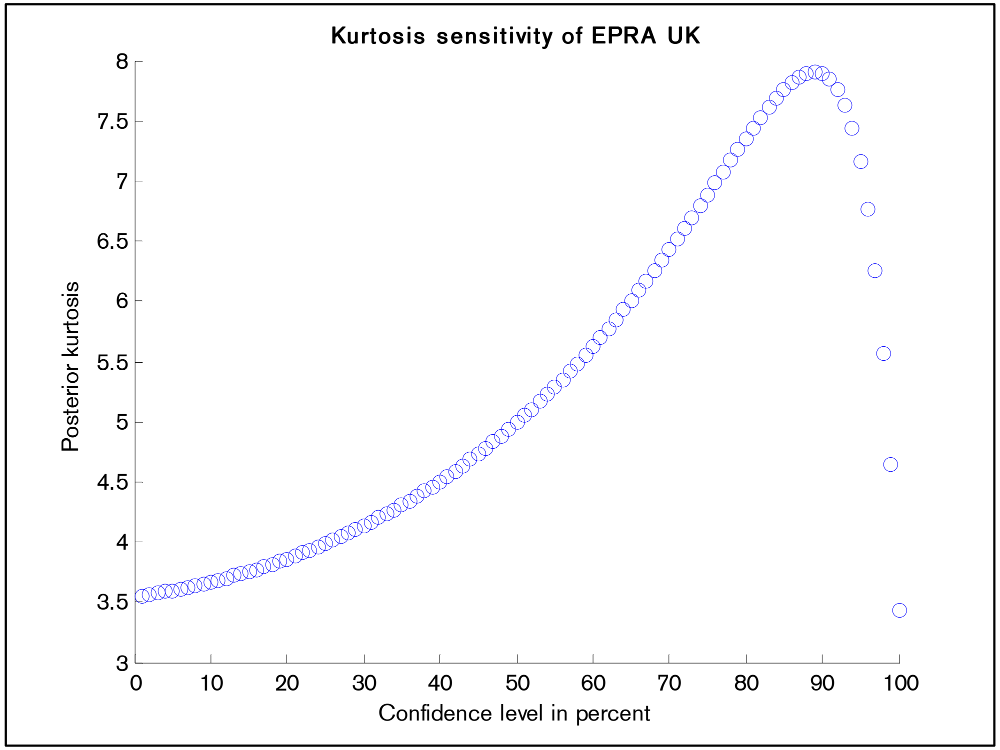
COP- Application
Assessment / Comparison

- Sensitivity of the skewness and the kurtosis of the posterior weekly return of the EPRA UK for different levels of confidence in our example.

Skewness sensitivity:



Kurtosis sensitivity:



Summary and Outlook
and Discussion…

- Using the COP approach, market data is pooled with the views that an investor has.
- A market posterior is derived, which may serve as input to a portfolio optimization, asset allocation decision, risk expectation building, pricing function……
- Portfolio optimization using the posterior market distribution can be done according to the respective needs, examples are mean-expected shortfall, VaR minimization with heavy tails, utility-based optimization…....
- Possible adjustments, extensions, shortcomings, caveats: form of the copula, fit of prior …?

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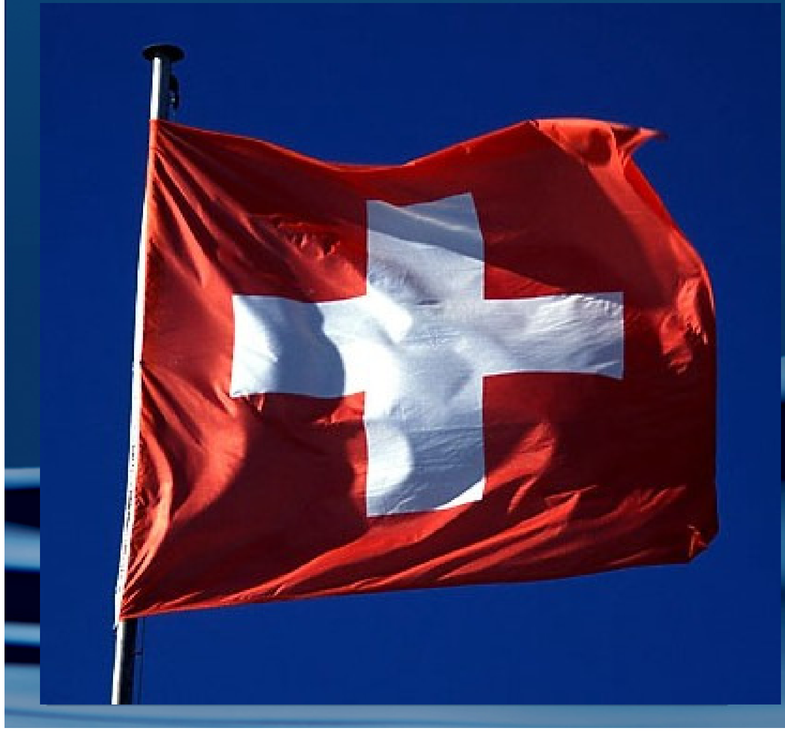
Thank you very much !!



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