**Topics in Labor Economics**

**Content:** This lecture is a topics course covering current research issues in labor economics. Prerequisites include basis microeconomic theory, intermediate econometrics and the lecture 'Labor Economics for Master/Diploma' or equivalent prior knowledge. The course has two parts. In the first part, there is a series of five lectures on static and dynamic labour supply models, human capital, labor demand, and search and matching theory. In the second part, participants will have to present and discuss critically a research paper or a policy report and to write a critical referee report on the chosen paper/report. The referee report has to be turned in on the day of the presentation. A particular emphasis in this course is on the interaction of theoretical and empirical modelling and its relevance for economic policy. The students in the course will learn to understand and critically discuss current research papers and reports in the area of labor economics. It is recommended for students to have a good background in economic theory and econometrics.

The assignment of the research papers to be discussed by the students take place during the first and second lecture.

**Time Schedule**

Five Lectures

1. Labor Supply (Thursday November 7, 10:00h – 14:00h)
2. Human Capital (Thursday November 14, 12:00h – 14:00h)
3. Labor Demand (Thursday November 28, 10:15h – 12:30h)
4. Search and Matching Theory (Thursday December 19, 10:30h – 14:00h)

The presentations of the research papers will take place on:

1. Wednesday 29 January 2013: 14:00-18:00h
2. Thursday 30 January 2013: 10:00-14:00h
All lectures and paper presentations take place in room 2330 (KG II).

References:


Section 1: Read the chapter by Blundell/MaCurdy in Ashenfelter/Card (1999) and Franz (2009, chapter 2)
Section 2: Read the chapter by Card in Ashenfelter/Card (1999) and Franz (2009, chapter 3)
Section 3: Read Franz (2009, chapter 4) and Cahuc/Zylberberg (2004, chapter 4) as well as
- The chapter by Autor and Acemoglu in Ashenfelter/Card (2011)
Section 4: Read Cahuc/Zylberberg (2004, chapter 3) as well as
- The chapter by Mortensen and Pissaridis in Ashenfelter/Card (1999)
Course Material – Section 1. Labor Supply

- Participation
- Home production
- Family labor supply

Dimensions of Labor Supply

- Quantity dimension: Number of persons supplying labor (head count)
- Behavioral dimension: Participation, hours
- Quality dimension: Heterogeneity in ability and education

Measures:

\[
\text{Participation rate} = \frac{\text{Employed Persons}}{\text{Population [in working age]}}
\]

\[
\text{Employment rate} = \frac{\text{Employees}}{\text{Population [in working age]}}
\]

\[
\text{Unemployment rate} = \frac{\text{Unemployed persons}}{\text{EMP} + \text{UNP}}
\]

\[
\text{Labor Supply} = \text{EMP} + \text{UNP}
\]
Static Models of Labor Supply

Reservation Wage versus Market Wage

Individual $i$:
- Reservation wage $w^R_i$
- Market wage $w_i$

Individual $i$ will offer $H_i$ hours of work, if $w_i \geq w^R_i$

Formally this means

$$H_i \begin{cases} = 0 & \text{for } w_i \leq w^R_i \\ > 0 & \text{for } w_i > w^R_i \end{cases}$$

$w_i$ and $w^R_i$ are determined by:

(1) $$w_i = X M_i \cdot \beta = \sum_{j=1}^{km} X M_{i,j} \cdot \beta_j$$

(2) $$w^R_i = X R_i \cdot \beta^R = \sum_{j=1}^{kR} X R_{i,j} \cdot \beta^R_j$$

where $X M_i$, $X R_i$ are determinants of $w_i$, $w^R_i$ (row vectors) and $\beta$, $\beta^R$ are column vectors of coefficients (magnitude of the influence), respectively

Then, the decision rule can be written as:

$$H_i \begin{cases} = 0 & \text{for } X M_i \cdot \beta \leq X R_i \cdot \beta^R \\ > 0 & \text{for } X M_i \cdot \beta > X R_i \cdot \beta^R \end{cases}$$

Assumption here: $w_i$ is exogenously predetermined

This assumption will be given up later

→ Investments in Human Capital
→ Search Theory

Here, we discuss the factors that influence $w^R_i$: opportunity costs of work

→ The value of leisure time
→ Other income
→ Situation in family
Participation and the Hours of Work as a Result of Individual Utility Maximization

- The labor supply is derived from the demand of leisure time as part of individual utility maximization problem → Consumer demand from microeconomics

Utility function:

\[ U = U(X, F, R, \mu) \]

\[ X: \text{consumption of goods} \]
\[ F: \text{leisure time} \]
\[ R: \text{observable individual characteristics} \]
\[ \mu: \text{unobservable individual characteristics} \]

Assumption: \( U \) is quasi-concave and twice differentiable

\[ U_X = \frac{\partial U}{\partial X} > 0, \quad U_F = \frac{\partial U}{\partial F} > 0, \quad U_{XX} = \frac{\partial^2 U}{\partial X^2} < 0, \quad U_{FF} = \frac{\partial^2 U}{\partial F^2} < 0 \]

- Positive but diminishing marginal utility

Constraints:

\[ F = T - H \]
\[ T: \text{Total disposable hours} \]
\[ H: \text{Hours of work} \]
\[ F \geq 0 \]
\[ H \geq 0 \]

\[ PX = W \cdot H + V \]
\[ P: \text{Price of consumption good (assumption of a representative good)} \]
\[ V: \text{Non-labor income} \]
\[ X \geq 0 \]

One can restrict analysis to the following two cases:

\[ H > 0 \quad \text{Participation} \]
\[ H = 0 \quad \text{No participation} \]
Kuhn–Tucker Problem

\[
\begin{align*}
\max & \quad U(X, T - H) \\
\text{s.t.} & \quad PX = wH + V \\
& \quad H \geq 0
\end{align*}
\]

Lagrange Function:

\[
L = U(X, T - H) - \lambda(PX - wH - V) + \kappa H; \quad \kappa > 0 \quad \text{for} \quad H = 0 \\
\kappa = 0 \quad \text{for} \quad H > 0
\]

First Order Conditions:

\[
(1) \quad \frac{\partial L}{\partial X} = U_X - \lambda P = 0 \\
(2) \quad \frac{\partial L}{\partial H} = -U_F + \lambda w \quad = 0 \quad \text{for} \quad H > 0 \\
\leq 0 \quad \text{for} \quad H = 0
\]
Approach: $U$ is quasi-concave

1) Solve (1) + (2) for $X$ and $H$
→ If solution results in $X, H > 0$, then there exists interior solution and individual supplies work.

2) Solve (1) for $H = 0$
→ If there is no solution under (1), then this is the solution.

Interior solution, $H > 0$:

It follows from (1)+(2) that

$$
\begin{align*}
U_X &= \lambda P \\
U_F &= \lambda w
\end{align*}
$$

\[ \frac{U_F}{U_X} \quad \frac{w}{P} \]

Marginal rate of substitution
Real wage

\[ \frac{U_F}{U_X} \geq \frac{w}{p} \]

i.e. if the marginal utility from leisure time relative to the marginal utility from consumption exceeds the real wage at zero hours already, then the individual doesn’t supply any labor.

The other way round, the marginal utility ratio at zero hours defines the real reservation wage, i.e. the minimal level of wage for a positive labor supply.

$$
\frac{w^R}{P} = \left. \frac{U_F}{U_X} \right|_{H=0} \quad \text{or} \quad w^R = P \cdot \left. \frac{U_F}{U_X} \right|_{H=0}
$$
Graphical Analysis

Individuum 1: Interior solution, i.e. $H_1 = T - F_1 > 0$

$$
\frac{U_1^F}{U_1^T}(X_1, T - H_1) = \frac{w}{P}
$$

Individuum 2: Doesn’t work. $H_2 = 0$  
$F_2 = T$  
$X_2 = \frac{V}{P}$

$$
\frac{U_2^F}{U_2^H}(X_2, T) > \frac{w}{P}  \iff w_2 > w
$$

**Reduced form** (solution of optimization problem depending on $W, P, V, R, \mu$):

$$
H = H(W, P, V, R, \mu) \quad \text{Labor supply}
$$

$$
X = X(W, P, V, R, \mu) \quad \text{Consumer demand}
$$

In addition, we have:

$$
F = T - H(W, P, V, R, \mu) \quad \text{Leisure demand}
$$

A central question is the reaction of labor supply in response to a change in wage rate, i.e. formally the derivative

$$
\frac{\partial H}{\partial W}
$$

Since $H$ is defined as $T - F$ from the leisure demand, you know from the household theory, that there exist income and substitution effects, with wage
being the price for leisure. For the interior solution \((H > 0)\), this is shown by the Slutsky decomposition:

\[
\frac{\partial H}{\partial w} = \left( \frac{H \partial H}{\partial V} \right)_{\text{Income effect (-)}} + \left( \frac{\partial H}{\partial w} \right)_{s}\n\]

here \(\frac{\partial F}{\partial V} > 0\) if leisure is not an inferior good

Graphical analysis of the effect of a wage increase on consumption and labor supply

- Here labor supply falls while wage rate increases
- The substitution effect \(H_1 \to H'\) \((H' > H_1)\)
- The income effect \(H' \to H_2\) \((H_2 < H')\)

It is easy to obtain this unexpected effect.
In contrast to the interior solution, the effect of a wage increase on the propensity to participate (= labor force participation rate) is unambiguous.

Assumptions:
\[ w_i^R = \bar{w}^R + \varepsilon_i \] Reservation wages differ across individuals

Probability of participation:
\[ P(w_i^R < w) = P(\bar{w}^R + \varepsilon_i < w) = P(\varepsilon_i < w - \bar{w}^R) = F(w - \bar{w}^R), \]
where \( F_\varepsilon(w - \bar{w}^R) \) distribution function of \( \varepsilon_i \)
\[ \frac{\partial P(w_i^R < w)}{\partial w} = f_\varepsilon(w - \bar{w}^R) \geq 0 \] probability density

If the appreciation of leisure increases as income grows (for example, leisure is a luxury good), then this can produce a backward bending labor supply function of the form
In macroeconomics, for simplification, often a constant labor supply is assumed

\[ \frac{\partial H}{\partial w} = H \frac{\partial H}{\partial V} + \left( \frac{\partial H}{\partial w} \right)_{u=\bar{u}} \cdot \frac{w}{H} \]

Slutsky decomposition

This can be transformed into elasticities

\[ \frac{\partial H}{\partial w} \cdot \frac{w}{H} = \left( \frac{\partial H}{\partial V} \cdot \frac{V}{\bar{H}} \right) \frac{wH}{V} + \left( \frac{\partial H}{\partial w} \cdot \frac{w}{\bar{H}} \right) \left. \right|_{u=\bar{u}} \]

where \( \eta_{HW} \) stands for uncompensated wage elasticity, \( \eta_{H,S} \) for compensated wage elasticity (compensated, because the income \( V \) is changed in a thought experiment so that utility stays the same) and \( \eta_{H,E} \) for income elasticity.
Slutzky Decomposition for Interior Solution

\[
(S) \quad \frac{\partial H(w, V)}{\partial W} = H \frac{\partial H}{\partial W} + \left( \frac{\partial H}{\partial W} \right)_s
\]

where \(w\) hourly wage, \(V\) nonlabor income, and \(H(w, V)\) Marshallian labor supply (based on demand for leisure).

Substitution effect: \(\frac{\partial H}{\partial W}_s > 0\)

Income effect: \(\frac{\partial F}{\partial V} > 0\) assuming leisure is a normal good \(\Leftrightarrow \frac{\partial H}{\partial V} < 0\)

Therefore sign of \(\frac{\partial H}{\partial W}\) is ambiguous

Note: If leisure is an inferior good \((\partial F/\partial V < 0 \Leftrightarrow \partial H/\partial V > 0)\), then unambiguously it holds \(\partial H/\partial W > 0\).

We show the comparative static result (S):

First order conditions for interior solution

\[\begin{align*}
(1a) \quad U_F - \lambda w &= 0 \\
(1b) \quad U_x - \lambda p &= 0 \\
(1) \quad pU_F - wU_x &= 0
\end{align*}\]

\[(2) \quad px - wH = V\]

\(\rightarrow\) Total differentiation of (1) and (2) with respect to \(x, H\) (these are the endogenous variables) and \(w, V\) (these are the exogenous variables), respectively

Differentiate with respect to \(x, H,\) and \(w:\)

\[\begin{align*}
(1') \quad -pU_{FF} dH + pU_{Fx} dx - U_x dW + wU_{xF} dH - wU_{xx} dx &= 0 \\
(2') \quad pdx - HdW - wdH &= 0
\end{align*}\]

In matrix notation:

\[
\begin{pmatrix}
-pU_{FF} + wU_{xF} & pU_{Fx} - wU_{xx} \\
-w & p
\end{pmatrix}
\begin{pmatrix}
dH \\
dW
\end{pmatrix}
= \begin{pmatrix}
U_x \\
H
\end{pmatrix}
\begin{pmatrix}
dx
\end{pmatrix}
\]

By Cramer’s rule, we have:

\[
\frac{dH}{dW} = \frac{\begin{vmatrix}
U_x & pU_{Fx} - wU_{xx} \\
H & p
\end{vmatrix}}{\begin{vmatrix}
-pU_{FF} + wU_{xF} & pU_{Fx} - wU_{xx} \\
-w & p
\end{vmatrix}} = \frac{pU_x - HpU_{Fx} + wH - wU_{xx}}{D}
\]

\(\rightarrow\) sign of \(dH/dW\) is ambiguous

Sign of denominator \(D\):

\[
D = -p^2 U_{FF} + wpU_{xF} + wpU_{Fx} - w^2 U_{xx}
= -(pw) \begin{pmatrix}
U_{FF} & U_{Fx} \\
U_{xF} & U_{xx}
\end{pmatrix} \begin{pmatrix}
w \\
p
\end{pmatrix} > 0
\]
since middle matrix is negative definite.

Now, it is necessary to determine the income effect: Total differentiation

\[
\begin{pmatrix}
-pU_{FF} + wU_{xF} & pU_{Fx} - wU_{xx} \\
-w & p
\end{pmatrix}
\begin{pmatrix}
\frac{dH}{dx} \\
\frac{dH}{dV}
\end{pmatrix} =
\begin{pmatrix}
0 \\
0
\end{pmatrix} dV
\]

\[
\frac{dH}{dV} = \begin{vmatrix}
0 & pU_{Fx} - wU_{xx} \\
1 & p
\end{vmatrix}
= \frac{-pU_{Fx} + wU_{xx}}{D}
\]

\(U_{Fx}\) is presumably positive \(\rightarrow\) Marginal utility of consumption increases with leisure

Leisure is no inferior good:
\[
\frac{dH}{dV} < 0 \iff \frac{dF}{dV} > 0 \quad \text{since} \quad dF = -dH
\]

Substitution effect:
\[
(1') \quad (-pU_{FF} + wU_{xF}) dH + (pU_{Fx} - wU_{xx}) dx = U_x dW
\]

and individual stays on a given indifference curve

\(3) \quad dU = -U_F dH + U_x dx = 0
\]

Solution by Cramer’s rule:
\[
\begin{vmatrix}
U_x & pU_{Fx} - wU_{xx} \\
U_x & 0
\end{vmatrix}
\begin{vmatrix}
U_x \\
U_x
\end{vmatrix}
= \frac{U_x (-pU_{FF} + wU_{xF}) + U_F (pU_{Fx} - wU_{xx})}{-p^2 U_{FF} + wpU_{xF} + \frac{U_F}{U_x} \cdot (p^2 U_{Fx} - wpU_{xx})}
\]

First order condition: \(\frac{U_F}{U_x} = \frac{w}{p}\)
\[
\begin{vmatrix}
\frac{dH}{dW}
\end{vmatrix}_s = \frac{pU_x}{-p^2 U_{FF} + wpU_{xF} + wpU_{Fx} - w^2 U_{xx}} = \frac{pU_x}{D} > 0
\]

Thus, Slutzky decomposition follows:
\[
\frac{dH}{dW} = \frac{pU_x}{D} + H \frac{wU_{xx} - pU_{Fx}}{D} = \left(\frac{dH}{dW}\right)_s + H \frac{dH}{dV}
\]

... and the derivation is completed!
**Taxation of Labor Income**: Less Clear Effects

**Linear income tax**

\[
\frac{w \cdot (1 - t) H + V(t)}{\tilde{w}} = P \cdot X \quad (t: \text{tax rate})
\]

\(\tilde{w}\): net wage (analysis as above)

\[
\frac{\partial H}{\partial t} = \frac{\partial H}{\partial \tilde{w}} \cdot \frac{\partial \tilde{w}}{\partial t} + \frac{\partial H}{\partial V} \cdot \frac{dV}{dt} = \left[ \left( \frac{\partial H}{\partial \tilde{w}} \right)_s + H \left( \frac{\partial H}{\partial V} \right) \right] (-w) + \frac{\partial H}{\partial V} \cdot \frac{dV}{dt}
\]

ambiguous

\(> 0\) if income effect dominates
\(< 0\) if substitution effect dominates

\[\Rightarrow\] As long as leisure is not an inferior good, also tax effect is ambiguous.

**Progressive Income Tax**

- more corner solutions are possible
- comparative statics require analysis of the discrete intervals (in a case-by-case analysis)

Empirical analysis is carried out using the concept of virtual income (non-labor income, \(V_i\))
\[ w_1 = (1 - t_1)w \]
\[ w_2 = (1 - t_2)w \]
\[ w_3 = (1 - t_3)w \]

\[ H_i = f(w_i, V_i) \]

⇒ Problem: kinks have to be modeled ⇒ clustering of observations at kinks to be expected
Non-convex budget constraint due to a high tax rate for welfare transfers (benefit withdrawal rate):

Cases to be distinguished: comparison of optimal interior/corner solutions in non-convex part

- Hours $H$ are not a continuous function of wage rate
- Observations clustering at the kink points is not observed in actual data

$\Rightarrow$ Differentiable Approximation of the Budget Constraint: MaCurdy et al. (1990)
Earned Income Tax Credit: EITC

Phase-out
Phase-in

Participation ↑
A, B, C: hours reduced
Home Production

\[ P = 1 \]

\[ h_1: \text{time in paid job} \]

\[ h_2: \text{time in home production} \]

\[
\begin{align*}
\max_{T, h_1, h_2, X} & \quad \underbrace{U(T - h_1 - h_2, X)}_{F} \\
\text{s.t.} & \quad X = f(h_2) + wh_1 \\
L = & \quad U(F, X) - \lambda[X - f(h_2) - wh_1]
\end{align*}
\]

(1) \[ \frac{\partial U}{\partial h_1} = -U_F + \lambda w = 0 \]

(2) \[ \frac{\partial U}{\partial h_2} = -U_F + \lambda f'(h_2) = 0 \]

(3) \[ \frac{\partial U}{\partial X} = U_x - \lambda = 0 \quad \iff \quad \lambda = U_X \]

(1') \[ U_F = U_X w \]

(2') \[ U_F = U_X f'(h_2) \]

\[ f'(h_2) = w \quad \Rightarrow \quad h_2^*(w) \]
\[ h_1^* = 0 \]
Intertemporal Labor Supply

\[
\max_{\{x(t), F(t)\}} V(0) = \sum_{t=0}^{K} (1 + s)^{-t} U[x(t), F(t)]
\]

s.t.: 

\[
A(0) + \sum_{t=0}^{K} (1 + r)^{-t} [w(t) H(t) - P(t) x(t)] = 0
\]

\[
T = H(t) + F(t)
\]

Lagrangean

\[
L = V(0) + \lambda \{ A(0) + \sum_{t=0}^{k} (1 + r)^{-t} [w(t) \cdot H(t) - P(t) x(t)] \}
\]

First order conditions:

(i) \((1 + s)^{-t} U_x(t) - \lambda (1 + r)^{-t} P(t) = 0\)

(ii) \((1 + s)^{-t} U_F(t) - \lambda (1 + r)^{-t} w(t) \geq 0\)

(iii) \(A(0) + \sum_{t=0}^{k} (1 + r)^{-t} [w(t) \cdot H(t) - P(t) x(t)] = 0\)

Reduced form solutions (interior solution for i + ii)

\[
H(t) = H[\lambda \theta^t w(t), \lambda \theta^t P(t)]
\]

\[
x(t) = x[\lambda \theta^t w(t), \lambda \theta^t P(t)]
\]

with \(\theta = \frac{1+s}{1+r}\) given \(\lambda\) Frisch demand.
\[ w_1(t) = w_0(t) + \bar{w} \]

\[ w_0(t) \rightarrow w_1(t) \]
\[ w_1^R(t) > w_0^R(t) \]

- Working life reduced form \( t_3 - t_8 \) to \( t_4 - t_5 \)
- Works more hours during \([t_5, t_6]\)
- Works less hours during \([t_4, t_5] + [t_6, t_7]\)
Empirical Specification of Participation Probability using a Probit Model

\[ I_i = \begin{cases} 
1 & \text{for } w_i > w_i^R : H_i > 0 \\
0 & \text{for } w_i \leq w_i^R : H_i = 0 
\end{cases} \]

\[ EI_i = Pr(w_i > w_i^R) \cdot 1 + Pr(w_i \leq w_i^R) \cdot 0 \]

\[ = Pr(w_i > w_i^R) \]

\[ = \text{Prob. of Participation} \]

Probit Model assumes:

\[ EI_i = Pr(H_i > 0) = \Phi(x_i\beta) = \Phi \left( \sum_{k=0}^{n} x_{ik}\beta_k \right) \]

with \( \Phi(z) \) distribution function of standard normal

\( x_{i1}, \ldots, x_{in} \) factors affecting participation

\[ \varphi(z) = \frac{d\Phi(z)}{dz} = \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}} \text{ density of a standard normal} \]

Effect of \( x_{ik} \) on participation probability

\[ \frac{\partial Pr(H_i > 0)}{\partial x_{ik}} = \beta_k\varphi(\sum_{k=1}^{n} x_{ik}\beta_k) = \beta_k\varphi(x_i\beta) \]

Maximum Likelihood Estimation

\[ \max_{\beta} L = \left( \prod_{i=1}^{N_1} \Phi(x_i\beta) \right) \cdot \left( \prod_{i=N_1+1}^{n} [1 - \Phi(x_i\beta)] \right) \]

\[ H_i > 0 \text{ for } i = 1, \ldots, N_1 \]

\[ H_i = 0 \text{ for } i = N_1 + 1, \ldots, N \]
Problem of Sample Selection Bias

$w_i$ only observable when $H_i > 0 : I_i = 1$
$w_i$ typically not known when $H_i = 0 : I_i = 0$

Estimation of coefficients $\gamma$ in

$$w_i = XM_i\gamma + v_i \quad i = 1, \ldots, N_1$$

is biased because based only on employees.
Employed : $I_i = 1$

with $E[v_i | I_i = 1, x_i] = \delta \cdot M(x_i\beta)$

joint normality $= \delta \cdot \phi(x_i\beta) / \Phi(x_i\beta) \neq 0$

$M(x_i\beta)$ is called the inverse Mills Ratio

Consistent estimation with selection correction term (Heckman Correction) $w_i = XM_i\gamma + \delta M(x_i\beta) + \tilde{v}_i$ based on estimated $\beta$.
$\Rightarrow$ yields consistent estimates for $\gamma$ and $\delta$.

Problem Endogeneity of Market Wage

(4) $w_i = XM_i\gamma + v_i$ Wage equation
(5) $H_i = \alpha w_i + x_i\theta + \varepsilon_i$ Hours equation if hours positive

Correlation between error terms $v_i$ and $\varepsilon_i$ implies that equation (5) cannot be estimated consistently, i.e. $\alpha$ and $\theta$ cannot be determined consistently.

Endogeneity of $w_i$ as explanatory variable in the hours equation.
Possible "solution" for the problems of selection bias and endogeneity of market wages

i) Based on sample of employees \((i = 1, \ldots, N_1)\) and the nonemployees \((i = N_1 + 1, \ldots, N)\) estimate \(\beta\) with a Probit estimator.

\[
(1) \quad \Pr(H_i > 0) = E(I_i) = \Phi(x_i\beta) \quad \text{and calculate} \quad M(x_i\beta) = \frac{\varphi(x_i\beta)}{\Phi(x_i\beta)}
\]

ii) Based on sample of employees \((i = 1, \ldots, N_1)\) estimate \(\gamma\) and \(\delta_w\) for given \(\beta\) from

\[
(2) \quad w_i = XM_i\gamma + \delta_w M(x_i\beta) + \tilde{v}_i \quad \text{and determine} \quad \hat{w}_i = XM_i\gamma
\]

iii) For employees estimate \(\alpha, \theta\) and \(\delta_h\) for given \(\gamma\) and \(\beta\)

\[
(3) \quad H_i = \alpha\hat{w}_i + x_i + \delta_h M(x_i\beta) + \varepsilon_i \quad \text{using a Tobit model.}
\]

When \((2) \langle \text{endogeneity of wages} \rangle\) is neglected then \((1)\) and \((3)\) can be summarized in a Tobit Model.
Which wage effect on hours is estimated?

\[ H_i = \alpha w_i + x_i \theta \]

- If \( x_i \) comprises non labor income \( V \), then

\[
\frac{\partial H_i}{\partial w_i} = \alpha \quad \text{Marshallian Effect} \quad \equiv \quad \text{uncompensated wage effect}
\]

\[
\frac{\partial H}{\partial V} = \theta_v \quad \text{Effect of change in non labor income}
\]

Total effect according to Slutzky decomposition

\[
\frac{\partial H_i}{\partial w_i} = \alpha = H_i \theta_v + \left( \frac{\partial H_i}{\partial w_i} \right)_s
\]

substitution effect (compensated wage effect)

Elasticities:

\[
\frac{\partial H_i w_i}{\partial w_i H_i} \equiv \eta_{H,w} = \varnothing_{H,E} \frac{w_i H_i}{V_i} + \eta_{H,S}
\]

Based on the estimates for \( \alpha \) and \( \frac{\partial H}{\partial V} \), the Slutzky decomposition allows to estimate the substitution effect

\[
\left( \frac{\partial H_i}{\partial w_i} \right)_s = \alpha - H_i \theta_v
\]

This provides a test for the static labor supply model because the theory unambiguously predicts that the substitution effect \( \left( \frac{\partial H_i}{\partial w_i} \right)_s \) should be positive.
Frisch Labor Supply under Uncertainty

Value Function Approach (assume $P_t = 1$)

$$V(A_t, t) \equiv \max_{\{X_t, F_t\}} \left\{ U(X_t, F_t) + \frac{1}{1+\delta} E_t V(A_{t+1}, t+1) \right\}$$

where

- $V(A_t, t) \equiv$ value (of current and future periods’ consumption) in period $t$
- $U(X_t, F_t) \equiv$ current period’s utility flow
- $E_t V(A_{t+1}, t+1) \equiv$ expected value of value in period $t+1$ given information in period $t$

s.t.: $A_{t+1} = (1 + r_{t+1})(A_t + W_t H_t - X_t)$

- $A_t$: state variable (financial assets in period $t$)
- Hours worked in period $t$: $H_t = T - F_t$

Why value function approach?

This approach allows to reduce the full intertemporal optimization problem into a sequence of current period decisions about this period and the future (the latter is captured by the value in $t+1$).

$\Rightarrow$ Bellmann Principle

- $A_t$: state variable
- $X_t, F_t, H_t$: decision variables
First order conditions:

Decision variables

\[
\frac{\partial V}{\partial F_t} = U_F(t) - \frac{1}{1+s} E_t \frac{\partial V}{\partial A_{t+1}} \cdot (1 + r_{t+1}) W_t = 0
\]

where \( \frac{\partial V}{\partial A_{t+1}} \equiv \lambda_{t+1} \) is the 'marginal utility of wealth'

\[
\Leftrightarrow \frac{U_F(t)}{W_t} = \frac{1}{1+s} E_t \lambda_{t+1}(1 + r_{t+1})
\]

where \( \lambda_{t+1}(1 + r_{t+1}) \) involves uncertainty about future (regarding \( r_{t+j} \) and \( W_{t+j} \) for \( j = 1, ... \))

\[
\frac{\partial V}{\partial X_t} = U_x(t) + \frac{1}{1+s} E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial x_t} = 0
\]

\[
\Leftrightarrow U_x(t) = \frac{1}{1+s} E_t \lambda_{t+1}(1 + r_{t+1})
\]

State variable: Decision variables respond to changes in \( A_t \)

\[
\frac{\partial V}{\partial A_t} = \lambda_t = \frac{\partial V}{\partial F_t} \cdot \frac{\partial F_t}{\partial A_t} + \frac{\partial V}{\partial X_t} \cdot \frac{\partial X_t}{\partial A_t} + \frac{1}{1+s} E_t \frac{\partial V}{\partial A_{t+1}} (1 + r_{t+1})
\]

Optimal decisions (interior solutions!) in \( F_t, X_t \) imply:

\[
\frac{\partial V}{\partial F_t} = \frac{\partial V}{\partial X_t} = 0
\]

[Note: a corner solution would imply: \( \frac{\partial V}{\partial F_t} \geq 0 \) for \( H_t = 0 \) ]

F.O.C. for the state variable \( A_t \) (interior solution):

\[
\frac{\partial V}{\partial A_t} = \lambda_t = \frac{1}{1+s} E_t \lambda_{t+1}(1 + r_{t+1})
\]
Take log’s and introduce prediction error

\[ \ln(\lambda_{t+1}) = \ln(\lambda_t) + b_{t+1}^* + \varepsilon_t^* \quad \text{(U)} \]

where

- \( b_{t+1}^* \) depends upon \((1 + s), (1 + r_{t+1})\), and the moments of \( \varepsilon_t^* \)
- \( \varepsilon_t^* \) error term with \( E_t \varepsilon_t^* = 0 \)

"Random Walk with drift in marginal utility"

Interpretation: Individual ”sets“ \( \lambda_0 \) at beginning of life cycle and then revises \( \lambda_t \) (”updates information on \( r_t \) and wages \( W_t \)) according to equation (U).
MaCurdy’s Approach To Model Intertemporal Labor Supply
Assuming Interior Solutions In All Periods

Period Specific Utility (individual $i$, period $t$):

$$U_i(x_{it}, h_{it}, A_{it}, \epsilon_{it}) = b_{it}x_{it}^{\gamma_i} - c_{it}\tilde{\gamma}h_{it}$$

where

- $x_{it}$ consumption and $h_{it}$ labor supply
- $\tilde{\gamma} = 1 + \frac{1}{\gamma}, \gamma > 0$, $\gamma$ is the intertemporal elasticity of substitution in labor supply, $0 < \gamma_1 < 1$ determines the intertemporal elasticity of substitution in consumption
- $b_{it}, c_{it}$ taste shifters

Specification of taste shifter (individual specific effect) for labor supply (disutility of work):

$$c_{it} = \exp\left[\frac{1}{\gamma}(-\beta A_{it} - \epsilon_{it})\right]$$

- $A_{it}$: observable characteristics
- $\epsilon_{it}$: unobservable characteristics

Note the additive separability between consumption $x_{it}$ and labor supply $h_{it}$ in the utility function. This implies that holding $\lambda$ constant ($\lambda$ is the Lagrange multiplier) the first order condition for labor supply (consumption) does not depend upon consumption (labor supply).

Define $\theta = \frac{1+s}{1+r}$ where $s$ discount rate and $r$ interest rate.

First order condition for labor supply $h_{it}$:

$$\frac{\partial U_i}{\partial F_{it}} = -\frac{\partial U_i}{\partial h_{it}} = -\lambda_{0it}\theta^t w_{it}$$

this defines Frisch labor supply where $\lambda_{0i}$ is the Lagrange Multiplier for individual $i$
From the first order condition, it follows

\[-c_{it} \tilde{\gamma} h_{it}^{-1} = -\lambda_{0i} \theta^t w_{it} \quad \Leftrightarrow \quad h_{it}^{-1} = \frac{\lambda_{0i} \theta^t w_{it}}{c_{it}}\]

\[\Leftrightarrow (\tilde{\gamma} - 1) \ln(h_{it}) = \ln(\lambda_{0i}) + t \ln(\theta) + \ln(w_{it}) - \ln(c_{it})\]

Note that \((\tilde{\gamma} - 1) = 1 + \frac{1}{\gamma} - 1 = \frac{1}{\gamma}\). Then, we have

\[\frac{1}{\gamma} \ln(h_{it}) = \ln(\lambda_{0i}) + t \ln(\theta) + \ln(w_{it}) - \frac{1}{\gamma}(-\beta A_{it} + \epsilon_{it})\]

As the final labor supply specification, one obtains

\[\ln(h_{it}) = \gamma \ln(\lambda_{0i}) + \gamma \ln(\theta) t + \gamma \ln(w_{it}) + \beta A_{it} + \epsilon_{it}\]

Note that \(\gamma \ln(\lambda_{0i})\) are the unobserved individual specific effects (fixed effects) introduced by MaCurdy into the panel analysis of labor supply.

\(\gamma (\ln(\theta)) t\) represents the age effects relative to \(\lambda_{0i}\) reflecting both the discounting of utility and the interest rate.

The above specification yields the Frisch labor supply elasticity

\[\frac{\partial \ln(h_{it})}{\partial \ln(w_{it})} = \gamma\]

which is the (\(\lambda\) constant) labor supply elasticity which represents the response to evolutionary wage changes.

**Econometric Analysis:**

i) Estimation in first differences for panel of individuals employed in all time periods (e.g. for prime-age males)

\[\Delta \ln(h_{it}) = \gamma \ln(\theta) + \gamma \Delta \ln(w_{it}) + \beta \Delta A_{it} + \Delta \epsilon_{it}\]

Instrument \(\Delta \ln(w_{it})\) because wage change may be endogenous.

This allows us to estimate the intertemporal elasticity of labor supply \(\gamma\) but does not provide estimates for the labor supply effects of parametric wage changes. For the latter, we also need to estimate the wage effects through \(\lambda_{0i}\), which is the next point.
ii) Modelling \( \lambda_{0i} \) as a function of exogenous characteristics and of the entire time path of endogenous variables

\[ \rightarrow \text{correlated random effects models (Chamberlain, MaCurdy)} \]

- For intertemporal labor supply, it is crucial to model \( \lambda_{0i} \) as a sufficient statistic for the life cycle effect.

- Thus, \( \lambda_{0i} \) captures the labor supply response to parametric wage changes, i.e. anticipated changes in wage profile.

Assume the following specification:

\[
\ln(\lambda_{0i}) = D_0 \varphi_0^* + \sum_{j=0}^T \gamma_{0j}^* E_0 \{\ln(w_{ij})\} + \theta_0^* A_0 + a_0^*
\]

where \( E_0 \) denotes expectation in period 0.

Define \( \varphi_0 = \gamma \varphi_0^* \), \( \gamma_{0j} = \gamma \gamma_{0j}^* \), and \( \theta_0 = \gamma \theta_0^* \). Then, the labor supply function is:

\[
\ln(h_{it}) = D_0 \varphi_0 + \sum_{j \neq t} \gamma_{0j} E_0 \{\ln(w_{ij})\} + \theta_0 A_0 + (\gamma + \gamma_{0t}) \ln(w_{it}) + \beta A_{it} + e_{it}
\]

with \( e_{it} = \epsilon_{it} + a_0 - \gamma_{0t} \left[ \ln(w_{it}) - E_0(\ln(w_{it})) \right] \)

Assuming for wage equation

\[
E_0 \{\ln(w_{it})\} = \pi_0 + \pi_1 t + \pi_2 t^2 + u_t
\]

Assume for property (interest) income (income accruing from initial wealth \( A_0 \))

\[
E_0 \{Y_{it}\} = \zeta_0 + \zeta_1 t + \zeta_2 t^2 + \eta_t
\]

to predict initial wealth \( A_0 \) as present value. Note that actual evolution of wealth is endogenized by the model (savings, interest income) and this is not part of property income!

Now, we have a specification of \( \ln(\lambda_{0i}) \) and we can estimate \( \gamma_{0t} \) (typically via IV estimation).

Based on the estimates, the response of \( \ln(h_{it}) \) to parametric wage change in period \( t \) is given by \( (\gamma + \gamma_{0t}) \).
Interpretation of Cross-sectional Econometric Labor Supply Specifications in light of Intertemporal Labor Supply Model:

- Studies often do not distinguish evolutionary and parametric wage changes when estimating the wage elasticity of labor supply.

- What is actually estimated depends upon whether the regressors (covariates) used explicitly or implicitly control for the life cycle effect, i.e. if proxies for the marginal utility of lifetime income are used.

Two Benchmark Examples:

i) A regression explaining hours of work as a function of current period wage uses all anticipated (age independent) determinants of wages over the lifecycle and preferences as additional regressors.
⇒ The wage elasticity estimated in this specification represents the Frisch Elasticity wrt evolutionary wage changes.

ii) Hours regression on property income (only referring to initial wealth), age, age squared, and log wage in current period yields reaction in response to parametric wage changes, i.e. the estimated wage elasticity combines the intertemporal elasticity of substitution $\gamma$ and the change in the individual specific effect ($\lambda_{0i}$: marginal utility of wealth).
⇒ Such a specification may use individual characteristics as instruments for the wage (i.e. the changes in the wage profiles)

Conclusion: Static regressions of labor supply typically do not fall into either of these two benchmark categories and the resulting estimated wage elasticities are often difficult to interpret in a life cycle model.
Basic Approaches in Family Labor Supply

Ref.: Blundell/MaCurdy (1999), section 7, Franz, Chapter 2.4

- Labor supply decisions are made in the context of the family

**Unitary model**

Extend the consumption leisure choice problem to include two leisure decisions

Family maximizes \( U(C, L_1, L_2, X) \)

where \( L_1, L_2 \) are hours of leisure for two family members
\( C \) family consumption (distribution does not matter)
\( X \) observable household characteristics

Constraints:

**Budget constraint** \( C = Y + W_1(T - L_1) + W_2(T - L_2) \)

**Time budget**
\( L_1 + H_1 = T \)
\( L_2 + H_2 = T \)

\( \lambda \) : Lagrange multiplier of budget constraint
Assume \( P_t = 1 \)

First Order Conditions:

\[
\frac{\partial U}{\partial L_1} \equiv U_{L_1} = \lambda W_1 \quad \text{and} \quad U_{L_2} \geq \lambda W_2
\]

\( U_C = \lambda \)

These conditions imply:

\[
U_{L_1} - U_C W_1 = 0
\]
\[
U_{L_1} - U_C W_2 \geq 0
\]
- Optimal labor supply choices in this framework satisfy the standard consumer demand restrictions of symmetry, negative semidefiniteness of Slutzky substitution matrix, and zero homogeneity in wages, prices, and nonlabor income.

- Symmetry requires equality between Slutzky cross-substitution terms

\[
\frac{\partial L_i}{\partial W_j} + L_j \frac{\partial L_i}{\partial Y} = \frac{\partial L_j}{\partial W_i} + L_i \frac{\partial L_j}{\partial Y} \quad \text{for} \quad i \neq j
\]

\[\left(\frac{\partial L_i}{\partial W_j}\right)_s = \left(\frac{\partial L_j}{\partial W_i}\right)_s\]

→ testable implications

Two regimes of working behavior:

i) both spouses participate:

\[H_1 = T - L_1 > 0 \quad \text{and} \quad H_2 = T - L_2 > 0\]

ii) individual 2 does not participate:

\[H_1 = T - L_1 > 0 \quad \text{and} \quad H_2 = T - L_2 = 0\]
Further implications (which are unlikely to hold!):

a) \( Y = Y_1 + Y_2 \) : Private unearned income \( Y_i \) received by individual \( i \) \((i = 1, 2)\)

Income pooling:

\[
\frac{\partial L_i}{\partial Y_1} = \frac{\partial L_i}{\partial Y_2} \quad i = 1, 2
\]

\( \rightarrow \) source of income does not matter

b) Non-participation of individual 2

\( U_{L_2} - U_C W_2 \geq 0 \)

\( \rightarrow \) It is the reservation wage of individual 2 rather than the market wage that affects marginally the labor supply decision of the partner, i.e. outside option value of paid work for a non-participant does not influence the allocation within the household. The latter would not hold in a strategic bargaining situation!
Collective model of family labor supply

- Basis for a lot of recent empirical work in labor supply effects of changes in tax/welfare policies
- Relaxing symmetry and income pooling, seeking instead solutions from efficient bargaining theory
- Basic premise: Individuals in family are a collection of individuals with their own utility function

Collective Framework:

\[
\max [\theta U_1 + (1 - \theta)U_2] \\
\text{s.t. } C_1 + C_2 + W_1 L_1 + W_2 L_2 = M
\]

- \(U_1, U_2\) utility of husband (1), wife (2)

Egoistic utility: \(U_1(C_1, L_1, X), U_2(C_2, L_2, X)\) are separate utilities of husband and wife

Caring utility: \(F_j[U_1(C_1, L_1, X), U_2(C_2, L_2, X)], j = 1, 2\) where \(F_j\) is total utility of \(j\)

→ Separability: \(L_2\) enters \(F_j\) only through \(U_2\) (private utility of wife) but no direct impact on husband
· Application of this model generates pareto-efficient outcomes:

→ goods privately consumed
→ no household production

\( \theta \) : utility weight of individual 1 with \( \theta = f(W_1, W_2, M) \)

Equivalent to sharing rule/decentralized solution:

- 1 gets income \( M - \varphi(W_1, W_2, X, M) \) and allocates income according to

\[
\max U_1 \quad s.t. \quad C_1 + W_1L_1 = M - \varphi(W_1, W_2, X, M)
\]

where \( \varphi(W_1, W_2, X, M) \) is defined as sharing rule

- 2 analogous

→ allocation in family depends upon relation wages and other variables in a way that reflects bargaining position of individuals

⇒ deviation from traditional marginal conditions possible
## Analogy between Human Capital and Physical Capital

<table>
<thead>
<tr>
<th></th>
<th>Physical Capital</th>
<th>Human Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment</strong></td>
<td>Buying Machines</td>
<td>Time spent in education / to educate plus monetary cost of educational materials / tuition ...</td>
</tr>
<tr>
<td><strong>“Return” ≡ direct effect</strong></td>
<td>Used as input factor in production of goods and services</td>
<td>Increases Labor productivity, Increases Marginal utility of Leisure / reduction of disutility of work</td>
</tr>
</tbody>
</table>
| **Interest rate (Return)** | Capital user costs (interest rate equals marginal productivity) | Wage $ W = RHK \times HK$
RHK= “Price of HK”
HK= “Amount of Human Capital” |
| **Depreciation** | Usage of physical capital for finite amount of time / limited amount of production; obsolescence when goods produced are not in demand any more | Forgetting Knowledge / Knowledge becomes obsolete when new production techniques are introduced |
| **Second hand market / Capital Loss** | Incomplete in most cases → can be sold at great loss in most cases | Human Capital cannot be “sold” since incorporated in person (Liquidation impossible) |
| **Amortization (Return period)** | Product cycle                           | Working life (focus on productivity effect)                                   |
Derivation of Mincerian earnings function based on Franz (2013, Chapter 3)

(6) \( Y_t = E_t - C_t \) with

\( Y_t \): Earnings in period \( t \)
\( E_t \): Potential earnings (\( \equiv \) all time devoted to work)
\( C_t \): Human Capital investment in period \( t \) (direkt costs and time spent)

Effect of human capital accumulation

(7) \[
E_t = E_{t-1} + r_{t-1}C_{t-1}
\]
with \( r_t \): rate of return

\[
= E_{t-1}(1 + r_{t-1}k_{t-1})
\]
\( k_{t-1} = \frac{C_{t-1}}{E_{t-1}} \): investment rate

This results in

(8) \[
E_t = E_0 \prod_{j=0}^{t-1}(1 + r_jk_j) \sim E_0 \times e^{\sum_{j=0}^{t-1}r_jk_j}
\]

Taking logs:

(9) \[
\ln(E_t) = \ln(E_0) + \sum_{j=0}^{t-1} r_jk_j
\]

\[
= \ln(E_0) + r_s \sum_{j=0}^{s-1} k_j + r_p \sum_{i=s}^{t-1} k_i
\]
with \( s \): years of schooling associated with \( r_s \)
\( t - s \): years of post school training with rate of return \( r_p \)

Mincer assumes: \( k_j = 1 \) for \( 0 \leq j \leq s - 1 \) schooling period
and \( k_i = k_s(1 - \frac{i-s}{T}) \) \( s \leq i \) post schooling period

\( k_s \): investments in first year of work
\( k_i \): declines linearly to zero at the end of working life \( (i = T + s) \)
For the schooling period: \( \sum_{j=0}^{s-1} k_j = s \)
Then, we have:

\[ (10) \ln E_t = \ln E_0 + r_s s + r_p \left[ k_s (t - s) \left( 1 - \frac{(t - s) - 1}{2T} \right) \right] \sum_{i=s}^{t-1} k_i \]

Only actual earnings are observed: \( Y_t = E_t (1 - k_t) \)

\[ (11) \ln(Y_t) = \ln(E_0) + r_s s + r_p [k_s (t - s) * (1 - \frac{(t - s) - 1}{2T})] + \ln(1 - k_t) \]

Use \( \ln(1 - k_t) \approx -k_t (1 + \frac{k_t}{2}) \) second order Taylor approximation

Then (11) can be rewritten as:

\[ (12) \ln(Y_t) = \left[ \ln E_0 - k_s \left( 1 + \frac{k_s}{2} \right) \right] + \frac{a}{r_s} s \]

\[ + \left[ r_p k_s \left( 1 + \frac{1}{2T} \right) + k_s \frac{1 + k_s}{T} \right] (t - s) \]

\[ - \left[ r_p \frac{k_s}{2T} + \frac{k_s^2}{2T^2} \right] (t - s)^2 \]

by collecting terms and plugging in \( k_t = k_s (1 - \frac{i - s}{T}) \)

In short hand notation this is

\[ \ln(Y_t) = a_0 + a_s + b_1 \text{Exp} + b_2 \text{Exp}^2 \]

with \( \text{Exp(erience)} = t - s, b_1 > 0, b_2 < 0 \)
Human Capital Formation

a) Earnings Functions

The causal effect of education on wages

Mincer Earnings Equation: $y_i$ earnings

$$\ln(y_i) = \alpha + \beta S_i + \gamma_1 E X_i + \gamma_2 E X_i^2 + \varepsilon_i$$

Human capital accumulation

$S_i$: Years of schooling (formal education) $\beta > 0$ returns to education

$E X_i$: Years of work experience (learning-by-doing, on-the-job training) $\gamma_1 > 0$ returns to experience $\gamma_2 < 0$ perience

• Very successful and often estimated regression

• Increasing wage differences between (and within) skill groups in various countries (US, UK, . . .)

• Society’s investment in human capital should depend on potential returns

$\rightarrow$ Causal interpretation of $\hat{\beta}, \hat{\gamma}_1, \hat{\gamma}_2$ ?

Problems (endogeneity of education and heterogeneity of returns to education):

- Ability Bias
- Self selection Bias
- Measurement error in education variables

Card’s (1999) human capital model with individual specific effects

Utility function of individual

\( U(S, y) = \ln(y) - h(S) \)

- \( S \): educational level (years of schooling)
- \( y(S) \): earnings level averaged over work phase
  \( \rightarrow \) being in the labor market: earnings increase with work experience
- \( h(S) \): convex cost function describing the disutility associated with an additional year of schooling
  \( (h'(S) = \text{marginal costs}) \)

Equation (1): condensed version of fully intertemporal problem
Special case: Discounted Present Value (DPV) Objective

\[ DPV = \int_{S}^{\infty} y(S) \exp(-rt) dt = y(S) \frac{\exp(-rS)}{r} \]

\[ \ln(DPV) = \ln(y(S)) - \underbrace{(rS + \ln(r))}_{h(S)} \]

Optimal Choice of S:

\[ \frac{\partial U}{\partial S} = \frac{y'(S)}{y(S)} - h'(S) = 0 \]

Special case of DPV:

\[ \frac{\partial \ln(y)}{\partial S} = \frac{y'(S)}{y(S)} = h'(S) = r \]

Interest rate

Return to education:

\[ \frac{\partial \ln(y)}{\partial S} > r \quad \text{Too little investment in human capital} \]

\[ \frac{\partial \ln(y)}{\partial S} < r \quad \text{Too much investment in human capital} \]

All the above assumes \( \log(y(S)) \) being concave

Individual specific effects:

- Heterogeneity in marginal costs \( h'(S) \)
- Heterogeneity in returns \( \frac{\partial \ln(y)}{\partial S} = \frac{y'(S)}{y(S)} \)
Simple specification discussed by Card:

\[ \frac{y'(S)}{y(S)} = b_i - k_1 S \]

(3)

\[ h'(S) = r_i + k_2 S \]

(4)

Note:
Only the levels \( b_i, r_i \) of the marginal effects differ by individuals \( i \)

Changes of marginal effects \( k_1 > 0, k_2 > 0 \) are constant across individuals \( i \).

Optimal choice of education:

\[ S^* = \frac{b_i - r_i}{k_1 + k_2} = \frac{b_i - r_i}{k} \quad \text{where} \quad k = k_1 + k_2 \]

(5)

Comparative Statics: \( \frac{\partial S^*}{\partial b_i} > 0 \) and \( \frac{\partial S^*}{\partial r_i} < 0 \)

i.e. individuals with a higher marginal return to \( S \) at a given \( S \) choose a higher level of \( S \). With the increase in \( S \), the marginal return to education decreases according to equation (3)

\[ \Rightarrow \text{Suppose } b_1 < b_2, \text{ then observed difference:} \]

\[ \frac{y'(S_2)}{y(S_2)} - \frac{y'(S_1)}{y(S_1)} < b_2 - b_1 \]
The specification in (3) is used in many empirical studies and it is typically assumed that $b_i$ is known when making the schooling decision.

Marginal return to education observed:

$$\beta_i = b_i - k_1 S^*_1 = b_i \left(1 - \frac{k_1}{k}\right) + r_i \frac{k_1}{k}$$

$\Rightarrow$ heterogeneous marginal returns across individuals depending both on $b_i$ and $r_i$ !!!

Special cases of constant marginal returns observed

$\beta_i = \text{constant}$, if either

a) $r_i = \bar{r}$ and $k_2 = 0$
   all individuals face the same constant marginal costs or

b) $b_i = \bar{b}$ and $k_1 = 0$
   all individuals face the same constant marginal returns

Distribution of returns is endogenous in general equilibrium

$\rightarrow$ higher supply of highly qualified workers should reduce the average marginal return

Assume that return schedules (3) are given from the viewpoint of one cohort of young individuals
Observed level of education and earnings

Equation (3) implies for individual \( i \)

\[
\ln(y_i) = \alpha_i + \beta_i S_i - \frac{1}{2} k_1 S_i^2
\]

where \( \alpha_i \) individual specific constant

Random Coefficient Model: Both intercept and slope differ by individuals Rewrite equation

\[
(6) \quad \ln(y_i) = a_0 + \bar{b} S_i - \frac{1}{2} k_1 S_i^2 + a_i + (b_i - \bar{b}) S_i
\]

will be part of error term

where \( a_i = \alpha_i - a_0 \), \( E(a_i) = 0 \) and \( E(b_i) = \bar{b} \)

- Equations (5) and (6) describe a two equation system determining \( S \) and \( y \) as functions of the random variables \( a_i, b_i, \) and \( r_i \)

- \( S_i^* \) is typically positively correlated with \( a_i \) and \( (b_i - \bar{b}) \)

\( \Rightarrow \) Traditional view: OLS regression on \( \ln(y_i) \) on \( S_i \) and \( S_i^2 \) overestimates the average return to education

- Ability bias: Positive correlation between \( a_i \) and \( S_i \)
- Self selection bias: Positive correlation between \( (b_i - \bar{b}) \) and \( S_i \)
Measurement Error:

- Error in reporting true educational level
- Differences in formally equal education levels (e.g. quality differences)
  \[ \Rightarrow \] Classical measurement error implies a downward bias in the OLS schooling coefficient
- Fixed Effects Estimation: Bias due to measurement error becomes more severe
- Measurement error and ability bias work in different directions
- If measurement error is negatively correlated with level of schooling (e.g. there exists a minimal and a maximal level of $S$ to be reported), then the bias is reduced
Alternatives to OLS

i) IV-Estimation

ii) Using family background variables

E.g. Twins: Assuming they have the same ability components $a_i$ and $b_i = \bar{b}$, then $\bar{b}$ can be consistently estimated by Within-Family-Differences specification

$$\ln(y_i) - \ln(y_j) = \bar{b}(S_i - S_j) - \frac{1}{2}k_{12}(S_i^2 - S_j^2)$$

where $a_i = a_j$ drops out ($i, j$: twins)

Problems:

- Where does observed difference $(S_i - S_j)$ come from? Measurement error will be important. Ability might not be perfectly correlated across family members.

- Family background variables are also typically no suitable instruments since these variables are correlated with unobserved ability component.

(This leads to IV-estimation)
ad i) IV-Estimation

Supply and Demand considerations suggest to use supply-side variables $Z_i$ as instruments for education where $Z_i$ affect the cost of education $r_i$ but $Z_i$ are uncorrelated with the unobserved ability components $a_i$ and $b_i \rightarrow \text{Card (2001) Econometrica}$

Take random coefficients model

$$
\ln(y_i) = a_0 + \bar{b}S_i - \frac{1}{2}k_1 S_i^2 + a_i + (b_i - \bar{b})S_i
$$

Assume

$$
r_i = Z_i \pi_1 + \eta_i
$$

where $E[\eta_i | Z_i] = E[a_i | Z_i] = E[(b_i - \bar{b}) | Z_i] = 0$ i.e. $Z_i$ affect $r_i$ and are uncorrelated to ability components $a_i$ and $(b_i - \bar{b})$

Choice of education:

$$
S_i^* = \frac{b_i - r_i}{k} = Z_i' \pi + \underbrace{\xi_i}_{= \frac{b_i - \bar{b} - \eta_i}{k}}
$$
If $Z_i$ is independent of $a_i$, $b_i - \bar{b}$, and $\xi_i$, then $\bar{b}$ can be estimated consistently with IV but not $a_0$ (with i.i.d. error terms)

$$E[S_i^2 | Z_i] = E[(Z_i\pi + \xi_i)^2 | Z_i]\]

$$ = (Z_i\pi)^2 + E[\xi_i^2 | Z_i] = \text{constant}$$

$$E[(b_i - \bar{b})S_i | Z_i] = E[(b_i - \bar{b})Z_i\pi | Z_i] + E[(b_i - \bar{b})\xi_i | Z_i] = 0 + E[\xi_i | Z_i] = \text{constant}$$

In general: Independence is a too strong assumption since a change in $Z_i$ affects the entire mapping between ability and schooling – even though the error terms are uncorrelated with $Z_i$ – leading to a systematic correlation between $(b_i - \bar{b})S_i$ and $Z_i$ !!!
• Orthogonality conditions are not sufficient to estimate average returns (average slope coefficients) in random coefficients model based on standard IV estimation

• Standard IV estimate

\[
\begin{align*}
(1) & \quad \hat{S}_i = Z_i \hat{\pi} \\
(2) & \quad \ln(y_i) = \beta_1 + \beta_2 \hat{S}_i + \beta_3 \hat{S}_i^2
\end{align*}
\]

• Estimates: \( \hat{\beta}_2, \hat{\beta}_3 \) in (2) \( \rightarrow \)

estimate reflects the effect of observed changes in instruments \( Z_i \) on education levels and the associated changes in earnings for those individuals who are induced to change their education level in reaction to the observed changes in \( Z_i \)

LATE: Local Average Treatment Effect
Wald–Estimator with Endogenous Dummy Variable  
(Important Special Case)

\[ Y_i = \beta_0 + \delta D_i + \varepsilon_i \quad \text{mit} \quad E\{\varepsilon_i|D_i\} \neq 0 \]

Coefficient \( \delta \) (=treatment effect) is constant  
and a dummy variable \( Z = 0, 1 \) is available as instrument

\textbf{Wald-IV-Estimator} on the basis of two samples \( \{Z = 0\} \) and \( \{Z = 1\} \)

\[ \hat{\delta} = \frac{\bar{Y}_{Z=1} - \bar{Y}_{Z=0}}{\hat{P}(D = 1|Z = 1) - \hat{P}(D = 1|Z = 0)} \]

where \( \hat{P}(D = 1|Z = j) = \frac{N_{1,j}}{N_j} \) ← number \( \{D = 1\} \) in group \( \{Z = j\} \) \( \quad \) \( \hat{N}_j \) ← size of group \( \{Z = j\} \)

\textbf{Wald’s Estimator} is equivalent to the 2SLS-estimator for this model  
(equation for \( D \) is a linear probability model)

\( \Rightarrow \) \( \text{IV-estimator relates induced (=exogenous) change of } Y \text{ by} \)

instrument variable \( (\bar{Y}_{Z=1} - \bar{Y}_{Z=0}) \) to the average change of  
the endogenous regressor \( E(D|Z = 1) - E(D|Z = 0) \).

\textbf{Local-Average-Treatment-Effect (LATE) Interpretation:}  
In the presence of heterogeneous effects of the dummy, the \( \text{IV-estimator identifies the average effect induced by variation of the instrument, i.e. the estimated parameter depends on the instrument (critique by Heckman, 1997, JHR).} \)
Extension: Control function approach

(1) \( \hat{S}_i = Z_i \hat{\pi} \) and \( \hat{\xi}_i = S_i - \hat{S}_i \)

(2) \( E[\ln(y_i)|S_i, Z_i] = a_0 + b\bar{S}_i - \frac{1}{2}k_1 S_i^2 + \lambda_1 \hat{\xi}_i + \psi_1 \hat{\xi}_i S_i \)

This is valid if the conditional expectations of \( a_i \) and \( b_i \) are linear in \( S_i \) and \( Z_i \)

\[
E(a_i|S_i, Z_i) = \lambda_1 S_i + \lambda_Z Z_i
\]

\[
E((b_i - \bar{b})|S_i, Z_i) = \psi_1 S_i + \psi_Z Z_i
\]

Equation (2) can be estimated consistently based on the residuals \( \hat{\xi}_i \) from the first stage regression of \( S_i \) on the instruments \( Z_i \)

\( \lambda_1 \neq 0 \) Ability bias

\( \psi_1 \neq 0 \) Self selection bias \( \rightarrow \) consistent estimate for \( \bar{b} \)
Generalization of Control Function Approach for Linear Regression

\[ Y_{1i} = Z_i' \pi + \varepsilon_{1i} \]
\[ Y_i = X_i' \gamma + \alpha_i Y_{1i} + \varepsilon_i \]
\[ = X_i' \gamma + \bar{\alpha} Y_{1i} + \varepsilon_i + (\alpha_i - \bar{\alpha}) Y_{1i} \]

Determination of \( E[\varepsilon_i + (\underbrace{\alpha_i - \bar{\alpha})}_{\eta_i}] Y_{1i} | Y_{1i}, Z_i] \)

\[ E\{\varepsilon_i|Y_{1i}, Z_i\} = E\{\varepsilon_i|\xi_{1i} = Y_{1i} - Z_i' \pi\} = \frac{Cov(\varepsilon_i, \xi_{1i})}{Var(\xi_{1i})} \cdot \varepsilon_{1i} \]
\[ E\{\eta_i|Y_{1i}, Z_i\} = \frac{Cov(\xi_{1i}, \eta_i)}{Var(\xi_{1i})} \cdot \varepsilon_{1i} \]

Two-stage approach:

1. Regression of \( Y_{1i} \) on \( Z_i \): \( \hat{\varepsilon}_{1i} = Y_{1i} - Z_i' \hat{\pi} \)
2. Regression: \( Y_i = X_i' \gamma + \bar{\alpha} Y_{1i} + \gamma_1 \hat{\varepsilon}_{1i} + \gamma_2 Y_{1i} \hat{\varepsilon}_{1i} + \tilde{\varepsilon}_i \)

\( \gamma_1 \neq 0 \): Endogeneity (Durbin-Wu-Hausman Test)

\( \gamma_2 \neq 0 \): Endogenous coefficients (\( \alpha_i \) are correlated with \( Y_{1i} \))
Special Case: $\gamma_2 = 0$ (no endogenous random coefficients)

$$Y_i = X_i'\gamma + \bar{\alpha}Y_{1i} + \gamma_1\hat{\varepsilon}_{1i} + \tilde{\varepsilon}_i$$

$$= X_i'\gamma + \bar{\alpha}(Z_i'\hat{\pi} + \hat{\varepsilon}_{1i}) + \gamma_1\hat{\varepsilon}_{1i} + \tilde{\varepsilon}_i$$

$$= X_i'\gamma + \bar{\alpha}Z_i'\hat{\pi} + (\bar{\alpha} + \gamma_1)\hat{\varepsilon}_{1i} + \tilde{\varepsilon}_i$$

asymptotically uncorrelated with $\hat{Y}_{1i}$

$\Rightarrow$ asymptotically equivalent to IV-estimation with $Z_i$ as instrument for $Y_{1i}$

General Case: Standard IV-estimation is inconsistent since in general

$$E[ (\alpha_i - \bar{\alpha}) \, Y_{1i} \mid Z_i] \neq 0,$$

because $Y_{1i}$ likely to be correlated with $\alpha_i$ (selective decision)

$$E[ (\alpha_i - \bar{\alpha}) \, Y_{1i} \mid Z_i] = E[ (\alpha_i - \bar{\alpha}) \, (Z_i'\pi + \varepsilon_{1i}) \mid Z_i]$$

$$= E[ (\alpha_i - \bar{\alpha}) \mid Z_i] \, Z_i'\pi + E[ (\alpha_i - \bar{\alpha}) \, \varepsilon_{1i} \mid Z_i]$$

$$= E (\alpha_i - \bar{\alpha}) \varepsilon_{1i} \neq 0 \text{ in general}$$
(i) Cost of labor: $W_i \cdot L_i$

- Not only wages and salaries, but also auxiliary labor costs, fluctuation costs, fringe benefits, costs of government regulation.

- Is the wage rate predetermined for a firm? Most likely, this is the case for contract (union) wages.

→ Additional benefits are not obligatory

Efficiency wage considerations

Monopsony on the labor market

What is Labor demand?

- Persons
- Hours

}  \quad \text{effects of reduction of working time (standard hours)}

Homogeneous labor is a fiction

(ii) Besides labor, other input factors are used

Here: Capital stock $K$ with the factor price $R$ (capital user costs)

- Substitution, when capital becomes more expensive  \quad \rightarrow \text{LD } \uparrow

- Scale effect, when capital becomes more expensive

→ production and therefore LD falls

(iii) Production technology $y = F(L, K)$

- Substitution possible between labor and other production factors

- Elasticity of substitution: describes extent by which firm can substitute labor for capital when wage rises

→ it depends also on technical progress (rationalization and capital deepening)
new jobs in other sectors
- Low and high skilled labor
→ complementarity between capital/technical progress and high skilled labor?

1. Production Technology and Market Structure
- LD of private firms: what is the profit maximizing input of labor L?

**Neoclassical production function:**

\[ y = F(L, K) \]

with \( F_L > 0, \quad F_{LL} < 0, \quad F_{LK} > 0 \)

**Capital** \( K \): capital user costs \( R(K) \)

**Labor** \( L \): remuneration costs per unit of labor \( W(L) \)

\[
\frac{\partial W(L)}{\partial L} > 0 \quad \text{... e.g. due to overtime premium}
\]

\[
\frac{\partial R(K)}{\partial K} > 0 \quad \text{... e.g. due to higher credit costs as a result of credit restrictions}
\]

**Commodity market:** conjectural demand function
- monopoly or
- monopolistic competition
→ it is crucial, that each price level is associated with a fixed amount of goods sold

Firm \( i \): demand for goods depends on \( P_i / \bar{P} \)

\( P_i \): price of the firm \( i \)

\( \bar{P} \): average price of the rivals

\( \alpha_i \): market share of the firm \( i \)
\[ \alpha_i(P_i/P) \text{ with } \frac{\partial \alpha_i}{\partial (P_i/P)} < 0 \]

Aggregate demand:
\[ \bar{y}(\bar{P}) \text{ with } \frac{\partial \bar{y}}{\partial \bar{P}} < 0 \]

Demand for goods from firm’s \( i \) perspective

\[ y_i = \alpha_i(P_i/P) \cdot \bar{y}(\bar{P}) = y_i(P_i, \bar{P}) \]  
\[ \eta^i_{y_i,P} = \frac{\partial y_i}{\partial P_i} \cdot \frac{P_i}{y_i} < 0, \quad \eta^i_{y,P} = \frac{\partial y_i}{\partial y} \cdot \frac{P}{y} < 0 \]

- The two elasticities, \( \eta^i_{y_i,P} = \eta^i_{y,P} \), are identical in monopoly case
  Note: in what follows, all elasticities are written as absolute values

- First we analyze variation of \( P_i \) at constant \( \bar{P} \)

Profit function for firm \( i \): (without index \( i \) except for \( \eta^i_{y,P} \))

\[ \Pi(L, K, y) = P(y) \cdot y - W(L) \cdot L - R(K) \cdot K - \lambda \cdot [y - F(L, K)] \]

- Production function as constraint

First order conditions FOC (differentiate with respect to \( L, K, y \)):

\[ \lambda \cdot F_L = W \cdot (1 + \eta_{W,L}) \]  
\[ \lambda \cdot F_K = R \cdot (1 + \eta_{R,K}) \]  
\[ \lambda = P \cdot \left(1 - \frac{1}{\eta^i_{y,P}} \right) \]

where \( \eta_{W,L} > 0, \eta_{R,K} > 0 \) are relative factor price changes associated with a change in required quantity of factor input

\( \lambda \): Lagrange multiplier shows increase in profit when production increases by one unit
Plugging in of (4.7) into (4.5)+(4.6) yields:

\begin{align*}
(4.8) & \quad P \cdot \left(1 - \frac{1}{\eta_{y,P}}\right) \quad F_L = W(1 + \eta_{W,L}) \\
(4.9) & \quad P \cdot \left(1 - \frac{1}{\eta_{y,P}}\right) \quad F_K = R(1 + \eta_{R,K})
\end{align*}

- These conditions represent the plausible formula

\[
\text{Marginal revenue} = \text{Marginal costs}
\]

where price variables are not constant as under perfect competition.

Marginal revenue = \[
\frac{\partial P(y)y}{\partial L} = \frac{\partial P(y)y}{\partial y} \cdot \frac{\partial y}{\partial L}
\]

\[= (P(y) + P'(y)y) \cdot F_L = P(y) \cdot \left(1 + \frac{P'(y) \cdot y}{P(y)}\right) \cdot F_L\]

\[= P(y) \cdot \left(1 + \frac{1}{y'(P) \cdot \frac{P}{y(P)}}\right) \cdot F_L\]

\[= P(y) \cdot \left(1 - \frac{1}{\eta_{y,P}}\right) \cdot F_L,
\]

where \(\eta_{y,P} = \left| \frac{y'(P)}{y(P)} \right| \cdot \frac{P}{y(P)}\)

Marginal costs = \[
\frac{\partial W(L) \cdot L}{\partial L} = W(L) + W'(L) \cdot L
\]

\[= W(L) \left(1 + \frac{W'(L) \cdot L}{W(L)}\right) = W(L) (1 + \eta_{W,L})\]
Under perfect competition:

\[ \eta_{W,L} = 0 \quad (\hat{=} \text{fixed wage}) \]

\[ \eta_{y,p}^i = \infty \quad (\hat{=} \text{fixed price level} \quad \hat{=} \text{horizontal demand curve}) \]

\[ \eta_{R,K} = 0 \quad (\hat{=} \text{fixed capital user costs}) \]

Then, we obtain the well known conditions:

\[ (4.8') \quad F_L = \frac{W}{P} \]

\[ (4.9') \quad F_K = \frac{R}{K} \]

For example: CES production function

\[ F(L, K) = y = \gamma \left[ \delta L^{-\rho} + (1 - \delta) K^{-\rho} \right]^{-\frac{\mu}{\rho}} \]

It follows from the FOC:

\[ \frac{K}{L} = \left( \frac{1 - \delta}{\delta} \cdot \frac{W}{R} \cdot \frac{1 + \eta_{W,L}}{1 + \eta_{R,K}} \right)^{\frac{1}{1+\rho}} \quad \text{does not depend upon}\ y! \]

When \( \eta_{W,L} \) and \( \eta_{R,K} \) are constant:

\[ \frac{d \ln \left( \frac{K}{L} \right)}{d \ln \left( \frac{W}{R} \right)} = \frac{1}{1 + \rho} =: \sigma \quad \text{Elasticity of substitution} \]

- CES: constant elasticity of substitution

\[ \rho = 0 \iff \sigma = 1 \]

- Cobb-Douglas:

\[ y = \gamma \cdot L^\mu \cdot K^{\mu(1-\delta)} \]
2. Factor price changes and labor demand

- For simplification:
  
  - Perfect competition in the input market ($W$ and $R$ exogenous)
  
  - Linear homogeneous production function

Reaction of LD in response to factor price changes

(i) substitution effect: firm uses less of the factor, which became comparatively more expensive
  
  → movement on an isoquant
  
  e.g.: if $W$ increases or $R$ declines, the firm uses less labor

(ii) scale effect:
  
  a) Rise in the factor price reduces the profit maximizing output and therefore the amount of input factors needed
  
  → rise in wages clearly causes a lower use of labor
  
  → rise in capital costs has this effect only in case, when scale effect overcompensates substitution effect
  
  b) the purchasing power effect arising from a wage increase shifts goods demand function to the right after substraction of scale effect losses
  
  → practically irrelevant in the micro-economic partial analysis

  Here, we abstract from the purchasing power effect of wage increase

We are interested in factor demand elasticities:

\[
\eta_{L,W} = \frac{\partial L}{\partial W} \cdot \frac{W}{L} \quad \text{and} \quad \eta_{L,R} = \frac{\partial L}{\partial R} \cdot \frac{R}{L}
\]
Derivation (unfortunately quite involved): Firm $i$

- In initial situation: production = sales (no storage)
  - Marginal Value Product (corrected for goods demand elasticities) matches factor price

\[
\begin{align*}
(1) & \quad y(P, \bar{P}) = F(L, K) \\
(2) & \quad W = F_L \cdot P \cdot \left(1 - \frac{1}{\eta_{y,p}^i}\right) \\
(3) & \quad R = F_K \cdot P \cdot \left(1 - \frac{1}{\eta_{y,p}^i}\right)
\end{align*}
\]

- if all firms react to factor price changes by adjusting their prices, the assumption of $\bar{P}$ being constant has to be given up

$\rightarrow$ for symmetric firms: $dP = d\bar{P}$

As a simplification, we can write then:

\[
\begin{align*}
(4) & \quad y(\lambda) = F(L, K) \quad \text{with} \quad \lambda = P \left(1 - \frac{1}{\eta_{y,p}^i}\right) \\
(5) & \quad W = F_L \cdot \lambda \\
(6) & \quad R = F_K \cdot \lambda
\end{align*}
\]

To determine the effect of a change in wages, equations (4)-(6) have to be differentiated with respect to $W$:

\[
\begin{align*}
(7) & \quad - \frac{\partial \lambda}{\partial W} \cdot \eta_{y,\lambda} \cdot \frac{y}{\lambda} = F_L \frac{\partial L}{\partial W} + F_K \frac{\partial K}{\partial W} \\
(8) & \quad 1 = F_L \frac{\partial \lambda}{\partial W} + \lambda \cdot \left(F_{LL} \frac{\partial L}{\partial W} + F_{LK} \frac{\partial K}{\partial W}\right)
\end{align*}
\]

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(9) \[ 0 = F_K \cdot \frac{\partial \lambda}{\partial W} + \lambda \cdot \left( F_{KL} \cdot \frac{\partial L}{\partial W} + F_{KK} \cdot \frac{\partial K}{\partial W} \right) \]

Because of results for linearly homogeneous production functions, we obtain

(7') \[ y \cdot \eta_{y,\lambda} \cdot \frac{\partial \lambda}{\partial W} + W \cdot \frac{\partial L}{\partial W} + R \cdot \frac{\partial K}{\partial W} = 0 \]

(8') \[ y \cdot \sigma \cdot \frac{\partial \lambda}{\partial W} - \frac{K}{L} \cdot R \cdot \frac{\partial L}{\partial W} + R \cdot \frac{\partial K}{\partial W} = \frac{y \cdot \lambda}{W} \sigma \]

(9') \[ y \cdot \sigma \cdot \frac{\partial \lambda}{\partial W} + W \cdot \frac{\partial L}{\partial W} - \frac{L}{K} \cdot W \cdot \frac{\partial K}{\partial W} = 0 \]

To determine \( \frac{\partial L}{\partial W} \), apply Cramer’s rule:

- Because it is very involved, I now present only the results (please check on your own!)

(10) \[ \frac{\partial L}{\partial W} = - \frac{L}{W} \left( \frac{L \cdot W}{y \cdot \lambda} \cdot \eta_{y,\lambda} + \frac{R \cdot K}{y \cdot \lambda} \sigma \right) \]

so that

(11) \[ \eta_{L,W} \equiv \frac{\partial L}{\partial W} \cdot \frac{W}{L} = - \left( \frac{L \cdot F_L}{y} \eta_{y,\lambda} \right) \left( \frac{K \cdot F_K}{y} \right) \cdot \sigma \]

Production elasticities correspond to shares:

\[
\frac{\partial \ln Y}{\partial \ln K} = s_K \quad \text{and} \quad \frac{\partial \ln Y}{\partial \ln L} = s_L
\]

This results in:

(12) \[ \eta_{L,W} = -s_L \eta_{y,\lambda} - (1 - s_L) \sigma \]

Under perfect competition:

(13) \[ \eta_{L,W} = -s_L \eta_{y,p} - (1 - s_L) \sigma \]
since $\eta_{y,\lambda} = \eta_{y,p}$ due to $\eta^i_{y,p} = \infty$

Again:

$$\eta_{L,W} = -s_L \eta_{y,p} - (1 - s_L)\sigma$$

scale effect  small if $s_L$ big
substitution effect  big if $s_L$ big

Note: Both terms are unambiguously negative

$\eta_{y,p}$: Sensitivity of aggregate demand with respect to the increase in prices in all firms

- Difference between (12) and (13):

$$\lambda = MR = P \left(1 - \frac{1}{\eta_{y,p}}\right)$$

is in (12) the basis for the firm’s decision making (monopolistic competition)

Marginal revenue

$\rightarrow$ labor demand effect in (12) is weaker because $MR$ increases at lower $y$

In an analogous manner to $\eta_{LW}$, one can derive $\eta_{L,R}$ (cross price elasticity) for perfect competition:

$$\eta_{L,R} = (1 - s_L) \cdot \left(\sigma - \eta_{y,p}\right)$$

- Reduction in capital user costs results in higher employment only if the scale effect dominates
It follows from (13) and (14):

\[
(15) \quad \eta_{LW} + \eta_{LR} = - (1 - s_L)\sigma + (1 - s_L)\sigma - s_L\eta_{y,p} + (1 - s_L)\eta_{y,p} = 0
\]

\[
= -\eta_{y,p} < 0
\]

- When \( W \) and \( R \) increase by the same relative amount (in percent), then only scale effect operates \( \Rightarrow \) employment falls

3. Profit maximization versus cost minimization

- Because of methodical considerations and in light of available data for empirical work: alternative illustration of production technology using duality theory based on cost function

\[
\begin{align*}
C &= R \cdot K + W \cdot L = \bar{C} \\
K &= \frac{\bar{C}}{R} - \frac{W}{R} L \\
\min C &= WL + RK \\
s.t.: y &= F(L, K) \\
\mathcal{L} &= WL + RK + \lambda[y - F(L, K)]
\end{align*}
\]

FOC:

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial L} &= W - \lambda F_L = 0 \quad \Rightarrow \quad \frac{W}{F_L} = \frac{F}{F_K} \\
\frac{\partial \mathcal{L}}{\partial K} &= R - \lambda F_K = 0
\end{align*}
\]
\[ \frac{\partial L}{\partial \lambda} = y - F(L, K) = 0 \]

- The first two FOC correspond to those under profit maximization in case of perfect competition on the factor markets
- The factor demand in case of cost minimization arises as a function of factor prices \((W, R)\) and output \(y\)

\[
\begin{aligned}
L &= L(W, R, y) \\
K &= K(W, R, y) \\
\end{aligned}
\]

zero homogeneous in factor prices

\[
C = C(L(W, R, y), K(W, R, y)) = C(W, R, y)
\]

According to Shephard’s Lemma:

\[
L = \frac{\partial C(W, R, y)}{\partial W} \quad K = \frac{\partial C(W, R, y)}{\partial R}
\]

\(\Rightarrow\) The cost function incorporates the same information about the technology as the production function (cost function and production function are dual to each other)

\(\Rightarrow\) These relationships are very important for the empirical analysis.

**Example:**

\[
C = y \cdot W^\alpha \cdot R^{1-\alpha}
\]

Applying Shephard’s Lemma:

\[
\begin{aligned}
L &= \frac{\partial C}{\partial W} = y \cdot \alpha \cdot W^{\alpha-1} \cdot R^{1-\alpha} = y \cdot \alpha \cdot \left( \frac{R}{W} \right)^{q-\alpha} \\
K &= \frac{\partial C}{\partial R} = y \cdot (1 - \alpha) \cdot W^\alpha \cdot R^{-\alpha} = y \cdot (1 - \alpha) \left( \frac{W}{R} \right)^{\alpha}
\end{aligned}
\]
Solving both equations for $\left( \frac{W}{R} \right)$:

$$\left( \frac{W}{R} \right)^{1-\alpha} = \frac{y^\alpha}{L} \iff W = \left( \frac{y^\alpha}{L} \right)^{\frac{1}{1-\alpha}}$$

$$\frac{W}{R} = \left( \frac{y(1 - \alpha)}{K} \right)^{-\frac{1}{\alpha}}$$

$$\left( \frac{y\alpha}{L} \right)^{\frac{1}{1-\alpha}} = \left( \frac{y(1 - \alpha)}{K} \right)^{-\frac{1}{\alpha}}$$

$$\iff y^{\frac{1}{1-\alpha}} + \frac{1}{\alpha} = \alpha^{\frac{1}{1-\alpha}}L^{\frac{1}{1-\alpha}} \cdot (1 - \alpha)^{-\frac{1}{\alpha}}K^{\frac{1}{\alpha}}$$

$$\iff y^{\frac{\alpha+1-\alpha}{1-\alpha}} = \alpha^{\frac{1}{1-\alpha}}(1 - \alpha)^{-\frac{1}{\alpha}}L^{\frac{1}{1-\alpha}}K^{\frac{1}{\alpha}}$$

$$\iff y = \alpha^{-\alpha}(1 - \alpha)^{-(1-\alpha)}L^{\alpha}K^{1-\alpha}$$

i.e. Cobb-Douglas cost function is dual to Cobb-Douglas production function.

General 2nd order approximation of cost function (Translog cost function):

$$\ln(C) = c_0 + a_1\ln(y) + 0.5a_2\ln(y)^2 + a_3\ln(y)\ln(W) + a_4\ln(y)\ln(R) + c_1\ln(W) + 0.5c_2\ln(W)^2 + c_3\ln(W)\ln(R) + 0.5c_4\ln(R)^2 + c_5\ln(R) + \beta_1t + \beta_2\ln(W)t + \beta_3\ln(R)t$$
Cost shares [derivatives of \(\ln(C)\) with respect to \(\ln(\text{factor price})\)] yield share equations:

\[
\begin{align*}
\frac{\partial \ln(C)}{\partial \ln(W)} &= \frac{\partial C}{\partial W} \cdot \frac{W}{C} = \frac{L \cdot W}{C} = s_L \\
\frac{\partial \ln(C)}{\partial \ln(R)} &= \frac{\partial C}{\partial R} \cdot \frac{R}{C} = \frac{R \cdot K}{C} = s_K
\end{align*}
\]

\[
\begin{align*}
s_L &= \frac{\partial \ln(C)}{\partial \ln(W)} = c_1 + c_2 \ln(W) + c_3 \ln(R) + \beta_2 t + a_3 \ln(y) \\
s_K &= \frac{\partial \ln(C)}{\partial \ln(R)} = c_5 + c_3 \ln(W) + c_4 \ln(R) + \beta_3 t + a_4 \ln(y)
\end{align*}
\]

- Zero homogeneity: \(c_2 + c_3 = 0\)

Adding up:

\[
\begin{align*}
&c_1 + c_5 = 1 & \beta_2 + \beta_3 = 0 \\
c_2 + c_3 = 0 & a_3 + a_4 = 0 & c_3 + c_4 = 0
\end{align*}
\]
• Neutral technical progress
  → \( \beta_1 < 0 \): factor input is falling over the time ceteris paribus

• Non-neutral technical progress
  \( \beta_3 > 0, \beta_2 < 0 \): technical progress favors demand for capital relative to that for labor (labor-saving)
  \( a_3 < 0, a_4 > 0 \): output growth favors relative demand for capital
  \( \beta_3 = \beta_2 = 0 \): neutral technical progress

• Wage elasticities given output:

  \[
  \frac{\partial \ln(L)}{\partial \ln(W)} = s_L \cdot \frac{c_2 + s_L^2 - s_L}{s_L^2} = \frac{c_2 + s_L^2 - s_L}{s_L} = \frac{c_2}{s_L} + (s_L - 1) < 0
  \]

  - Elasticity of substitution:

  \[
  \frac{c_3 + s_L \cdot s_K}{s_L s_K} = \frac{c_3}{s_L s_K} + 1
  \]

  This is an important approach in empirical work
4. Heterogeneous labor

- Homogeneous production factor labor implausible
  $\rightarrow$ different dimensions

- Current economic policy debate about skill bias in labor demand
  with the demand for low skilled labor falling relative to high
  skilled labor

$L_U$: low skilled labor

$L_H$: high skilled labor

K: Capital

Hypotheses:

1) Complementarity between capital and high skilled labor

\[
\frac{\partial \ln L_U}{\partial \ln R} = s_K \cdot \sigma_{UK} > 0 \quad \text{substitutes}
\]

\[
\frac{\partial \ln L_H}{\partial \ln R} = s_K \cdot \sigma_{HK} < 0 \quad \text{complements}
\]

If there are only two input factors, they can be only substitutes

2) Non neutral technical progress

\[
\frac{\partial \ln \left( \frac{L_U}{L_H} \right)}{\partial t} < 0 \quad \text{i.e. falling relative demand for}
\]

factor prices, $y$

low skilled labor over the time

Heterogeneous demand for labor, skill biased technological change (SBTC), globalization, and institutions


Stylized view: The increase in earnings inequality in the UK/US and the rise in unemployment rates in continental Europe during the 80s and 90s reflect an increase in the relative demand for more highly skilled labor since the relative supply of more highly skilled labor has increased at the same time.

Framework mostly used to study these developments: Supply, demand, and institutions SDI

Two skill groups: S skilled
U unskilled

\(N_S, N_U\) employment
\(w_S, w_U\) wages
Change from period 0 to period 1:

- Inelastic relative supply increases from $N_0$ to $N_1$
- Labor demand increases from $D_0$ to $D_1$

We observe

- higher skill differential in wages $w_1$ in period 1 compared to $w_0$ in period 0 and
- higher relative employment of skilled workers $N_1$ in period 1 relative to $N_0$ in period 0
→ The increase in demand at a given wage ratio was larger than the increase in supply

⇒ Search for reasons why relative demand has changed: SBTC, change in product demand across industries with different skill intensities e.g. through increasing international trade

Institutional effects:
Intersection of supply and demand curves describes competitive equilibrium
→ Rents for groups of workers can change over time
→ Minimum wages (e.g. through unions) can prevent wage adjustment in equilibrium: If wage remains at old level $w_0$, the relative demand $\frac{N_S}{N_U}$ exceeds the relative supply resulting in unemployment of the unskilled workers
Formalization based on a CES-Produktion Function

Output: Constant returns to scale

\[ Q_t = \left[ \alpha_t (a_t N_{st})^\rho + (1 - \alpha_t) (b_t N_{ut})^\rho \right]^{\frac{1}{\rho}} \]

\( N_{st}, N_{ut} \): skilled/unskilled employment in period \( t \)

\( \sigma = \frac{1}{1-\rho} \): elasticity of substitution and \( \rho \leq 1 \iff \sigma > 0 \)

\( \alpha_t \): share of activity assigned to skilled employment

\( a_t, b_t \): skilled/unskilled labor augmenting technical progress

Skill biased technical change (SBTC)

Increases in \( \frac{a_t}{b_t} \) or \( \alpha_t \)

\( \frac{a_t}{b_t} \uparrow \) intensive SBTC: skilled workers become relatively better at existing jobs

\( \alpha_t \uparrow \) extensive SBTC: "upskilling" of work tasks
If both input factors are paid by their marginal products

\[ \frac{w_{st}}{w_{ut}} = \frac{\alpha_t}{1 - \alpha_t} \cdot \left( \frac{a_t}{b_t} \right)^\rho \cdot \left( \frac{N_{st}}{N_{ut}} \right)^{-\frac{1}{\sigma}} \]

In logarithms:

\[ \ln \left( \frac{w_{st}}{w_{ut}} \right) = \ln \left( \frac{\alpha_t}{1 - \alpha_t} \right) + \rho \ln \left( \frac{a_t}{b_t} \right) - \frac{1}{\sigma} \ln \left( \frac{N_{st}}{N_{ut}} \right) \]

This equation explains the skill differential in wages as the result of relative factor supplies (× inverse of elasticity of substitution) and technical progress shifting relative labor demand. Only for \( \rho > 0 \) (i.e. \( \sigma > 1 \)) does a skill bias in intensive technical progress \( \left( \frac{a_t}{b_t} \uparrow \right) \) yield a higher skill differential in wages.