1. (8.2)
Use theorem 4.14 and its corollary to show that if \(X_{11}, X_{12}, \ldots, X_{1n_1}, X_{21}, X_{22}, \ldots, X_{2n_2}\) are independent random variables, with the first \(n_1\) constituting a random sample from an infinite population with the mean \(\mu_1\) and the variance \(\sigma_1^2\) and the other \(n_2\) constituting a random sample from an infinite population with the mean \(\mu_2\) and the variance \(\sigma_2^2\), then
   
a) \[ E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2; \]
   
b) \[ \text{var}(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}. \]

2. (8.47)
Find the sampling distribution of \(Y_1\) and \(Y_n\) for random samples of size \(n\) from a population having the beta distribution with \(\alpha = 3\) and \(\beta = 2\).

3. (10.2)
If \(\hat{\Theta}_1\) and \(\hat{\Theta}_2\) are unbiased estimators of the same parameter \(\theta\), what condition must be imposed on the constants \(k_1\) and \(k_2\) so that \(k_1\hat{\Theta}_1 + k_2\hat{\Theta}_2\) is also an unbiased estimator of \(\theta\)?

4. (10.7)
Show that \(\frac{X + 1}{n + 2}\) is a biased estimator of the binomial parameter \(\theta\). Is this estimator asymptotically unbiased?

5. (10.52)
Given a random sample of size \(n\) from a uniform population with \(\alpha = 0\), find an estimator for \(\beta\) by the method of moments.

6. (11.2)
If \(x_1\) and \(x_2\) are the values of a random sample of size 2 from a population having a uniform density with \(\alpha = 0\) and \(\beta = \theta\), find \(k\) so that \(0 < \theta < k(x_1 + x_2)\) is the \((1 - \alpha)100\%\) confidence interval for \(\theta\) when
   
a) \(\alpha \leq 1/2;\)
   
b) \(\alpha > 1/2.\)

7. (11.33)
Independent random samples of size \(n_1 = 16\) and \(n_2 = 25\) from normal population with \(\sigma_1 = 4.8\) and \(\sigma_2 = 3.5\) have the means \(\bar{x}_1 = 18.2\) and \(\bar{x}_2 = 23.4\). Find a 90% confidence interval for \(\mu_1 - \mu_2\).
8. (12.6)  
A single observation of a random variable having an exponential distribution is used to test the null hypothesis that the mean of the distribution is $\theta = 2$ against the alternative that it is $\theta = 5$. If the null hypothesis is accepted if and only if the observed value of the random variable is less than 3, find the probabilities of type I and type II errors.

9. (12.7)  
Let $X_1$ and $X_2$ constitute a random sample from a normal population with $\sigma^2 = 1$. If the null hypothesis $\mu = \mu_0$ is to be rejected in favor of the alternative hypothesis $\mu = \mu_1 > \mu_0$ when $\bar{X} > \mu_0 + 1$, what is the size of a critical region?

10. (12.13)  
With reference to Exercise 12.12, if $n = 100$, $\theta_0 = 0.40$, $\theta_1 = 0.30$, and $\alpha$ is as large as possible without exceeding 0.05, use the normal approximation to the binomial distribution to find the probability of committing a type II error.

Exercise 12.12  
Use the Neyman-Pearson lemma to indicate how to construct the most powerful critical region of size $\alpha$ to test the null hypothesis $\theta = \theta_0$, where $\theta$ is the parameter of a binomial distribution with a given value of $n$, against the alternative hypothesis $\theta = \theta_1 < \theta_0$.

11. (13.47)  
Nine determinations of the specific heat of iron had a standard deviation of 0.0086. Assuming that these determinations constitute a random sample from a normal population, test the null hypothesis $\sigma < 0.01$ at the 0.05 level of significance.

12. (13.75)  
Samples of an experimental material are produced by three different prototype processes and tested for compliance to a strength standard. If the tests showed the following results, can it be said at the 0.01 level of significance that the three processes have the same probability of passing this strength standard?

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<th>Process A</th>
<th>Process B</th>
<th>Process C</th>
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<td>58</td>
<td>49</td>
</tr>
<tr>
<td>Number failing test</td>
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<td>35</td>
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</table>