Chapter 8

8.2 Use Theorem 4.14 and its corollary to show that if \( X_{11}, X_{12}, \ldots, X_{1n_1}, X_{21}, X_{22}, \ldots, X_{2n_2} \) are independent variables, with the first \( n_1 \) constituting a random sample from an infinite population with the mean \( \mu_1 \) and the variance \( \sigma_1^2 \) and the other \( n_2 \) constituting a random sample from an infinite population with the mean \( \mu_2 \) and the variance \( \sigma_2^2 \), then

(a) \[ E(\overline{X}_1 - \overline{X}_2) = \mu_1 - \mu_2; \]
(b) \[ \text{var}(\overline{X}_1 - \overline{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}. \]

8.59 Integrate the appropriate chi-square density to find the probability that the variance of a random sample of size 5 from a normal distribution with \( \sigma^2=25 \) will fall between 20 and 30.

8.70 Find the sampling distributions of \( Y_1 \) and \( Y_n \) for random samples of size \( n \) from a population having the beta distribution with \( \alpha=3 \) and \( \beta=2 \).