Repeat Exam – Financial Data Analysis
at the University of Freiburg
(Winter Semester 2008/2009)

Friday, January 15, 2009, 8.00–9.30am
Part 1 (46 Points)

Consider the following model for log returns $r_t$:

$$ r_t = c + \epsilon_t - \theta_1 \epsilon_{t-1}, $$

where $\epsilon_t$ is a white noise term with $E(\epsilon_t) = 0$ and $E(\epsilon_t^2) = \sigma^2$.

a) (4 pts) Derive $r_{t+1}^* = E[r_{t+1} | I_t]$ with $I_t = \{r_t, r_{t-1}, \ldots\}$. Interpret the obtained quantity. [Hint: $E(\epsilon_t | I_t) = \epsilon_t$.]

b) (10 pts) Show that model (1) can be transformed into the following form:

$$ r_t = \phi_0 - \sum_{i=1}^{\infty} \phi_i r_{t-i} + \epsilon_t. $$

What are the necessary restriction(s) on the parameter(s) in order to obtain this transformation? Use this transformation to obtain the conditional expectation $E[r_{t+1} | I_t]$ with $I_t = \{r_t, r_{t-1}, \ldots\}$. Use this result together with that of part a) to represent $E(\epsilon_t | I_t)$ as an explicit function of $\{r_t, r_{t-1}, \ldots\}$.

c) (6 pts) Derive the unconditional mean $E[r_t]$ and the unconditional variance $\text{Var}[r_t]$ under model (1). Is $r_t$ under model (1) weakly stationary? Why?

d) (6 pts) Derive the autocorrelation function $\{\rho_l\}_{l=1,2,3,\ldots}$ of $r_t$ under model (1).

e) (14 pts) Table 1 in the Appendix provides the results ($p$-values) of the Ljung-Box test and the Runs-test for the daily log returns of IBM as well as for the residuals of model (1) fitted to the IBM log return data. Do these results confirm model (1)? Explain your answer.

f) (6 pts) Assume that $\epsilon_t \sim \text{iid} N(0, \sigma^2)$. Derive the log-likelihood function for $\{r_t\}_{t=1}^T$ under model (1) and under the (initial) condition $\epsilon_0 = 0$. (6pts) [Hint: if $X \sim N(\mu, \sigma^2)$, the p.d.f. of $X$ is given by $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(x-\mu)^2}{2\sigma^2} \right\}$.]
Part 2 (16 Points)

Consider the following ARCH(1) model:

$$\epsilon_t = \sigma_t u_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2$$

where

$$u_t \sim \text{iid} N(0, 1), \quad \alpha_0 > 0, \quad 0 \leq \alpha_1 < 1.$$ 

a) (4 pts) Show that under the ARCH(1) model the variable $\epsilon_t^2$ follows an AR(1)-process.

b) (2 pts) Derive the unconditional variance of $\epsilon_t$.

c) (2 pts) Why is it necessary to impose the above restrictions on the parameters?

d) (2 pts) Derive the formula for the one-step ahead forecast at time period $T$ for the conditional variance under model (3), i.e. $\sigma_T^2(1)$.

e) Table 2 in the Appendix reports the results of the Jarque-Bera-test applied to the standardized residuals $\hat{u}_t = \epsilon_t / \hat{\sigma}_t$ of model (3).

i. (2 pts) Which hypothesis is tested by the Jarque-Bera-test?

ii. (2 pts) What is the expected value of the skewness and kurtosis of $u_t$ under model (3)?

iii. (2 pts) What could you conclude from the results of Table 2?
Consider the following ordered probit model for the transaction price change data of stocks:

\[
 y_i = \begin{cases} 
 s_1, & \text{if } \alpha_0 < y_i^* \leq \alpha_1 \\
 s_2, & \text{if } \alpha_1 < y_i^* \leq \alpha_2 \\
 s_3, & \text{if } \alpha_2 < y_i^* < \alpha_3 
\end{cases}
\]

\[
y_i^* = \text{TDIFF}_i \beta_1 + y_{i-1} \beta_2 + y_{i-2} \beta_3 + y_{i-3} \beta_4 + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2),
\]

where

- \( y_i^* \) : unobservable continuous price change at time \( t_i, i = 1, \ldots, n \);
- \( y_i \) : observable discrete price change at time \( t_i \) with three possible categories;
- \( y_{i-l} \) : observable discrete price change at time \( t_{i-l}, l = 1, 2, 3 \);
- \( \text{TDIFF}_i \) : \( t_i - t_{i-1} \), time elapsed between trade \( i-1 \) and \( i \), measured in seconds.

a) (2 pts) What is the motivation to assume for transaction price changes such an ordered probit model?

b) (8 pts) What signs would the Roll-model (for the bid-ask bounce) predict for the coefficients \( \beta_2, \beta_3, \beta_4 \) in model (4)? Explain your answer briefly.

c) (4 pts) Table 3 in the Appendix contains the estimation results from EVIEWS for the IBM stock using the ordered probit model described above. What are the estimated values of \( (\alpha_0, \alpha_1, \alpha_2, \alpha_3) \)? Give a brief interpretation of those estimates.

d) (4 pts) You do not find the estimated value of \( \sigma \) in the EVIEWS outputs, why?

e) (10 pts) Consider the following statement: "If the elapsed time between two trades increases, the probability that the observed price change falls into the categories of \( s_1 \) would be (ceteris paribus) higher." Comment on this statement based on the results from Table 3.
Appendix

Table 1: Results ($p$-value) of the Ljung-Box (LB) test and the Runs-test for the daily log returns of IBM and for the residuals of the fitted MA(1) model.

<table>
<thead>
<tr>
<th>Series</th>
<th>LB(5) - Test</th>
<th>LB(10) - Test</th>
<th>Runs - Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p$-value</td>
<td>$p$-value</td>
<td>$p$-value</td>
</tr>
<tr>
<td>Log returns</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Residuals of MA(1)</td>
<td>0.35</td>
<td>0.17</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 2: EVIEW output of residual test.

Series: Standardized Residuals
Sample: 25220
Observations: 5219

Mean: -0.022501
Median: -0.033820
Maximum: 5.701424
Minimum: -11.71274
Std. Dev.: 0.998606
Skewness: -0.296529
Kurtosis: 9.333445
Jarque-Bera: 8799.287
Probability: 0.000000
Table 3:

Estimation results from EVIEWS for the IBM stock using the ordered probit model described in Part 3.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDIFF</td>
<td>-0.000985</td>
<td>-7.147931</td>
<td>0.0000</td>
</tr>
<tr>
<td>Y(-1)</td>
<td>-1.120639</td>
<td>-101.3027</td>
<td>0.0000</td>
</tr>
<tr>
<td>Y(-2)</td>
<td>-0.569018</td>
<td>-50.73320</td>
<td>0.0000</td>
</tr>
<tr>
<td>Y(-3)</td>
<td>-0.225282</td>
<td>-22.23194</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Limit Points

| LIMIT_0:C(5) | -1.142969 | -139.9096 | 0.0000 |
| LIMIT_1:C(6) | 1.088048  | 134.4753  | 0.0000 |

Pseudo R-squared 0.124022  Akaike info criterion 1.523553
Schwarz criterion 1.524577  Log likelihood -39558.40
Hannan-Quinn criter. 1.523873  Restr. log likelihood -45159.12
LR statistic 11201.44  Avg. log likelihood -0.761661
Prob(LR statistic) 0.000000