Exercise Session - Problem Set 1

Problem 1

For the matrices

\[
A = \begin{pmatrix} 1 & -3 & 5 \\ 2 & -2 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 7 & 2 \\ 1 & -3 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 2 & -1 \\ 4 & -9 & -3 \\ 5 & 2 & 3 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 7 & 5 \\ 11 & 8 & 2 \end{pmatrix},
\]

compute i) \(AB\) ii) \(BA\) iii) \(A'B'\) iv) \(AC\) v) \(CA\) vi) \(C-D\) vii) \(A+D\).

Problem 2 (Based on Wooldridge, Example D.14, p. 818)

a) Use the properties of trace to prove that for any \(n \times m\) matrix \(\operatorname{tr}(A'A) = \operatorname{tr}(AA')\).

b) Verify that \(\operatorname{tr}(A'A) = \operatorname{tr}(AA')\) using matrix \(A\) in Problem 1.

Problem 3

For the matrix

\[
A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}
\]

calculate \(|A|, \operatorname{tr}(A), A^{-1}\).

Problem 4

Write the following system of linear equations in matrix notation and solve for the vector \(x\). Under which assumption a nonhomogeneous system of equations will have an unique solution?

\[
\begin{align*}
x_1 - x_2 + x_3 &= 2 \\
x_1 + x_2 - x_3 &= 0 \\
-x_1 - x_2 - x_3 &= 6
\end{align*}
\]

Problem 5 (Based on Greene, p.12–15 and Example 5, p. 59)

a) Express the sum of the elements in any \(n \times 1\) vector \(x\) in matrix terms. Do the same for the case where all elements in \(x\) are equal to the same constant \(a \neq 0\).

b) Express the arithmetic mean for any \(n \times 1\) vector \(x\).

c) A fundamental matrix in econometrics is the idempotent matrix \(M^0\), which is used to form deviations from sample average. For the \(n \times 1\) vector \(x\) derive \(M^0\). What are the properties of idempotent matrices?
d) Express the sum of deviations about the mean for the $n \times 1$ vector $\mathbf{x}$ using $\mathbf{M}^0$. Express also the sum of squared deviations about the mean in matrix terms.

e) Prove that for $K \times 1$ column vectors, $\mathbf{x}_i, i = 1, ..., n$, and some nonzero vector $\mathbf{a}$,

$$
\sum_{i=1}^{n} (\mathbf{x}_i - \mathbf{a})' (\mathbf{x}_i - \mathbf{a})' = \mathbf{X}' \mathbf{M}^0 \mathbf{X} + n(\bar{x} - \mathbf{a})' (\bar{x} - \mathbf{a})',
$$

where the $ith$ row of $\mathbf{X}$ is $\mathbf{x}_i'$ and $\mathbf{M}^0$ is the idempotent matrix defined in c).

[Hint: For the solution of a)–d) use a $n \times 1$ vector of ones denoted by $\mathbf{j}_n$]