Section 4. Static Labor Demand

Reference: Borjas, Chapter 4

Production function

\[ q = f\left(\underbrace{E}_{\text{employees in persons}}, \underbrace{K}_{\text{capital}}\right) \]  \hspace{1cm} (1)

Marginal product

\[ MP_E = \frac{\partial q}{\partial E} \]
\[ MP_K = \frac{\partial q}{\partial K} \]

Average product \( \triangleq \text{productivity} \)

\[ AP_E = \frac{q}{E} \]
\[ AP_K = \frac{q}{K} \]

Production function for \( K = \text{constant} \):
Profit maximization

\[(2) \quad \Pi = pq - wE - rK \]

for \(w, r, p\) given \(\rightarrow\) perfectly competitive firm

Value marginal product

\[VMP_E = p \cdot MP_E\]

Value average product

\[VAP_E = p \cdot AP_E\]

Max \(\Pi\) yields First Order Condition (FOC):

\[\frac{\partial \Pi}{\partial E} = p \cdot MP_E - w = 0\]

\[\Leftrightarrow VMP_E = p \cdot MP_E \equiv w\]

Second Order Condition: \(VMP_E\) declining i.e. \(E > \overline{E}^*\) max of \(MP_E\)

Capital\(\equiv\)constant: Labor Demand in the short run

No adjustment in capital implies short run demand curve:

\[E^{SR} \left( \frac{w}{p} \right) = MP_E^{-1} \left( \frac{w}{p} \right)\]
→ actually inverse of \( MP_E \) function in real wage provided price is constant

Elasticity of labor demand in short run

\[
\delta_{SR} = \frac{\partial E_{SR}}{\partial \Delta w} \sim \frac{\Delta E_{SR}}{E_{SR}} \cdot \frac{w}{E_{SR}}
\]

Employment decision in the long run:

→ Long-run labor demand when the firm’s capital stock is not fixed

Isoquants: \( f(E, K) = \bar{q} \) → combinations of \( E \) and \( K \) resulting in the same level of output \( \bar{q} \)

\[ q_1 > q_0 \]

Slope of isoquant:

\[
\frac{dK}{dE} = \frac{-\partial f}{\partial E} = \frac{-MP_E}{MP_K}
\]

→ marginal rate of technical substitution
Profit maximization implies cost minimization

**Iso cost curve:** \( C = wE + rK \) with \( r \): price of capital

\( \rightarrow \) combinations of \( E \) and \( K \) resulting in the same costs \( C \)

\[
K = \frac{C}{r} - \frac{w}{r} E
\]

Cost minimization yields combination \( (E, K) \) in \( P \) with \( \frac{w}{r} = \frac{MP_E}{MP_K} \)

or put differently \( \frac{MP_E}{w} = \frac{MP_K}{r} \)

i.e. 'get the same additional output for each Euro.'

or put differently \( \frac{w}{MP_E} = \frac{r}{MP_K} \)

i.e. 'pay the same for an additional unit of output produced by using more labor or by using more capital'.
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Profit maximization:

\[
\max_{\{E, K\}} \pi = p q - w E - r K \quad \text{given } w, r, p
\]
\[
\text{s.t. } q = f(E, K)
\]

\[
\begin{align*}
\text{FOC: } & \frac{\partial \pi}{\partial E} = p MPE - w = 0 & \iff & w = p MPE \\
& \frac{\partial \pi}{\partial K} = p MPK - r = 0 & \iff & r = p MPK
\end{align*}
\]

conditions for profit maximization

Profit max: \( p = MC \) (marginal cost) \( \Rightarrow \frac{w}{r} = \frac{MPE}{MPK} \)

implies condition for cost minimization

Thus, profit maximization \( \Rightarrow \) cost minimization

Long-run demand for labor

- How does employment react in response to a fall in the wage \( \rightarrow \) fall in wage causes fall in MC

Increase from \( E_0 \) to \( E_1 \) can be decomposed into scale and substitution effect
- Scale effect results from increase in \( q \): \( q_0 \rightarrow q_1 \)
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It is clear that

\[
\frac{\partial E}{\partial w} < 0
\]

because both scale effect and substitution effect go into the same direction

\[
\frac{\partial K}{\partial w}
\]

because \( \left( \frac{\partial K}{\partial w} \right)_{\text{subst.}} > 0 \) and \( \left( \frac{\partial K}{\partial w} \right)_{\text{scale}} < 0 \)

The long run labor demand curve is steeper than the short run

- General principle in economics: agents can respond more easily to changes when facing fewer constraints (in short run cannot change capital input!)
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Elasticity of Substitution: $\sigma$

Isoquants

Perfect substitutes

\[
\sigma = \infty
\]

Substitution possible but becomes increasingly difficult

\[
0 < \sigma < \infty
\]

Perfect complements

(limitational technology)

\[
\sigma = 0
\]

(Cobb Douglas: $\sigma = 1$)

\[
\sigma = \frac{\partial K}{\partial MP_E} \cdot \frac{E \cdot MP_E}{K \cdot MP_K} \equiv \frac{\text{Percent change in } (\frac{K}{E})}{\text{Percent change in } (\frac{w}{r})}
\]

because

\[
\frac{MP_E}{MP_K} = \frac{w}{r}
\]

by cost minimization

The larger $\sigma$ is, the larger is the substitution effect
Scale effect depends on output demand

Let \( \eta \) be elasticity of output demand

\[
\frac{\partial q^d}{\partial p} \cdot \frac{p}{q} = \eta < 0 \quad \text{monopolistic competition}
\]

then long run labor demand elasticity is (under constant returns to scale)

\[
\eta_w^E = \frac{\partial E}{\partial w} \cdot \frac{w}{E} = \eta s_E - \sigma (1 - s_E) < 0
\]

scale effect substitution effect

where \( s_E = \frac{wE}{wE + rK} \equiv \text{labor share in costs} \)

This formula, which is generally true, reflects Marshall’s rules of derived demand:

Labor demand is more elastic, the greater

- the elasticity of substitution \( \sigma \)
- the elasticity of demand for output \( |\eta| \) in absolute value
- labor’s share in total costs
- the supply elasticities of other factors of production, such as capital
Aside: Linear homogeneous production function

\[ y = F(E, K) \Rightarrow \lambda y = F(\lambda E, \lambda K) \]

Euler Theorem:

\[ 1 \cdot y = E \cdot F_E + K \cdot F_K \quad \text{(Adding up)} \]

\[ \iff 1 = \frac{E \cdot F_E}{y} + \frac{K \cdot F_K}{y} = s_L + s_K \]

(scale elasticity of production)

Define: \( F_E = \frac{\partial F}{\partial E} \) and \( F_K = \frac{\partial F}{\partial K} \)

If \( F(E, K) \) is linear homogeneous, then \( F_E(E, K) \) and \( F_K(E, K) \) are homogeneous with degree zero (zero homogeneous). This means

\[
F_E(E, K) = F_E(\lambda E, \lambda K) \\
F_K(E, K) = F_K(\lambda E, \lambda K)
\]

Analogous to the Euler Theorem, the following holds also for zero homogeneous functions:

\[
0 = 0 \cdot F_E(E, K) = L \cdot F_{LL}(E, K) + K \cdot F_{EK}(E, K) \\
0 = 0 \cdot F_K(E, K) = L \cdot F_{KL}(E, K) + K \cdot F_{KK}(E, K)
\]

This implies:

\[
F_{EE} = -\frac{K}{E} \cdot F_{EK} \quad \text{and} \quad F_{KK} = -\frac{E}{K} \cdot F_{KE} = -\frac{E}{K} \cdot F_{EK}
\]

Finally, the elasticity of substitution is:

\[
\sigma = \frac{d \ln \left( \frac{K}{E} \right)}{d \ln \left( \frac{F_E}{F_K} \right)} = \frac{F_E \cdot F_K}{y \cdot F_{EK}}
\]
Equilibrium in the Labor Market and Minimum Wages

Macro Perspective: Homogeneous labor, Equilibrium in the labor market

A statuatory (obligatory) minimum wage applies to the whole economy.
More realistic case: Minimum wage only applies for part of the economy (this is the covered sector). In the other part (uncovered sector), there is wage flexibility, so there will be adjustment to the labor market equilibrium in this sector.

- In Sector 1, the wage increases and employment decreases.
- Therefore, labor supply increases in sector 2 and wage decreases and employment increases
  ⇒ Marginal product in sector 2 is smaller than in sector 1, that means the welfare loss in sector 1 is bigger than the welfare increase in sector 2.
Monopsony in the labor market

Also, with market power of the firm in the labor market (here single monopsony), profit maximization implies

\[
\text{Value marginal product} = \text{Marginal cost of labor}
\]

i.e.

\[
P \cdot q_E = w \cdot (1 + \eta_w, E) > w
\]

\begin{itemize}
  \item Introduction of a minimum wage \( w_m \) between \( w_0 \) and \( \overline{w}_m \) increases the employment \( \Rightarrow \) employment under minimum wage \( E^*_m > E_0 \)
    since marginal cost of labor is now equal to \( w_m \) and not \( w \cdot (1 + \eta_w, E) = \overline{w}_m \) at \( E_0 \).
  \item Reasons, why employment effects of minimum wages may not be negative or even positive \( \Rightarrow \) Card and Krueger „Myth and Measurement“, 1994.
\end{itemize}