Problem 1

For the matrices
\[
A = \begin{pmatrix}
1 & -3 & 5 \\
2 & -2 & 0
\end{pmatrix}
B = \begin{pmatrix}
7 & 2 \\
1 & -3 \\
2 & 4
\end{pmatrix}
C = \begin{pmatrix}
0 & 2 & -1 \\
4 & -9 & -3 \\
5 & 2 & 3
\end{pmatrix}
D = \begin{pmatrix}
1 & 7 & 5 \\
11 & 8 & 2 \\
2 & 4 & 0
\end{pmatrix},
\]
compute i) \(AB\) ii) \(BA\) iii) \(A'B'\) iv) \(AC\) v) \(CA\) vi) \(C-D\) vii) \(A+D\).

Problem 2 (Based on Problem D.4, p. 798)

a) Use the properties of trace to prove that for any \(n \times m\) matrix \(tr(A'A) = tr(AA')\).

b) Verify that \(tr(A'A) = tr(AA')\) using matrix A in Problem 1.

Problem 3

For the matrix
\[
A = \begin{pmatrix}
1 & 3 & 3 \\
1 & 4 & 3 \\
1 & 3 & 4
\end{pmatrix}
\]
calculate \(|A|\), \(tr(A)\), \(A^{-1}\).

Problem 4

Write the following system of linear equations in matrix notation and solve for the vector \(x\). Under which assumption a nonhomogeneous system of equations will have an unique solution?
\[
\begin{align*}
x_1 - x_2 + x_3 &= 2 \\
x_1 + x_2 - x_3 &= 0 \\
-x_1 - x_2 - x_3 &= 6
\end{align*}
\]

Problem 5 (Based on Problem D.7, p. 798)

Let \(A\) be an \(n \times n\) symmetric, positive definite matrix. Show that if \(P\) is any \(n \times n\) nonsingular matrix, then \(P'AP\) is positive definite.