Review Exercises (Session by Dr. Kestl)

Solution to Question 3:

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u \]

Suppose \( x_3 \) is the omitted variable yielding the regression model

\[ y = \tilde{\beta}_0 + \tilde{\beta}_1 x_1 + \tilde{\beta}_2 x_2 + u \]

Knowing that

\[ \tilde{\beta}_1 = \frac{\sum_{i=1}^{n} \hat{r}_{i1} y_i}{\sum \hat{r}_{i1}^2} \]

where \( \hat{r}_{i1} \) is the residuals from regressing \( x_{i1} \) on \( x_{i2} \),

then

\[ \tilde{\beta}_1 = \frac{1}{\sum \hat{r}_{i1}^2} \sum \hat{r}_{i1} \left( \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + u_i \right) \]

\[ \tilde{\beta}_1 = \frac{1}{\sum \hat{r}_{i1}^2} \left\{ \beta_0 \sum \hat{r}_{i1} + \beta_1 \sum \hat{r}_{i1} x_{i1} + \beta_2 \sum \hat{r}_{i1} x_{i2} + \beta_3 \sum \hat{r}_{i1} x_{i3} + \sum \hat{r}_{i1} u_i \right\} \]

\[ \tilde{\beta}_1 = \frac{1}{\sum \hat{r}_{i1}^2} \left\{ \beta_1 \sum \hat{r}_{i1} x_{i1} + \beta_3 \sum \hat{r}_{i1} x_{i3} + \sum \hat{r}_{i1} u_i \right\} \]

\[ \hat{x}_i = \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \hat{\beta}_3 x_{i3} \]

\[ \Sigma \hat{r}_{i1}^2 = \Sigma \hat{r}_{i1} x_{i1} = \Sigma \hat{r}_{i1} x_{i3} = \Sigma \hat{r}_{i1} u_i \]

\[ \mathbb{E} [\tilde{\beta}_1] = \frac{1}{\sum \hat{r}_{i1}^2} \beta_1 \frac{\sum \hat{r}_{i1}^2}{\sum \hat{r}_{i1}^2} + \beta_3 \frac{\sum \hat{r}_{i1} x_{i3}}{\sum \hat{r}_{i1}^2} + \frac{\sum \hat{r}_{i1} E[ u_i ]}{\sum \hat{r}_{i1}^2} \]

\[ \mathbb{E} [\tilde{\beta}_1] = \beta_1 + \beta_3 \frac{\sum \hat{r}_{i1} x_{i3}}{\sum \hat{r}_{i1}^2} \]