Empirical Market Microstructure Analysis (EMMA)


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Outline

1. Introduction: Financial Markets and Market Structure
2. **Financial Market Equilibrium Theory and Asset Pricing Models**
3. Statistical Building Blocks and Econometric Basics
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Relation asset pricing models with market microstructure models

• In this lecture we focus on general aspects of financial market equilibria. The lecture is based heavily on chapter 2 of the book by DeJong and Rindi. It provides the theoretical basis for the papers and concepts discussed later in the course.

• Consider first the arbitrage pricing principle that holds in both complete and incomplete markets:
  • If the payoffs from a financial security can be replicated by a some portfolio build of other securities, then the price of the security must be exactly identical to the price of such a portfolio.

• The arbitrage pricing theory (APT) for financial market assets is originated in Ross (1976). In the APT however, factor structures are used to determine an asset’s price. Factors enter the pricing equation through sensitivities, called factor loadings.
Relation asset pricing models with market microstructure models

- Related to the APT is the CAPM, which is using a market portfolio to estimate an asset’s price, with the sensitivity of the asset’s return regarding market returns corresponding to one factor only. It was developed by Sharpe (1964), Lintner (1965) and Mossin (1966) with origination in Treynor (1961).

- The two models are the most influential original models for asset pricing, with other important models being the consumption based capital asset pricing model, the Fama-French 3 factor model, the Carhart 4 factor model....
Relation asset pricing models with market microstructure models

- As models for asset pricing like other models rely on several assumptions, those may be more or less strict and realistic.
- Some models in the field of financial market microstructure share several assumptions of those models, but many assumptions will be relaxed to take into account how agents behave in markets.
- Especially the role of information and how information is observed, used and revealed is crucial for the way the markets work and how prices are derived.
- The role of information with respect to decision-making will be obvious in the following derivations of financial market equilibria.
- Generally, the models laid out below are related to standard microeconomic models with utility maximizations under assumptions and restrictions given by the prevailing structures at hand.
Allocation mechanism and information signal

• Prices play a central role regarding the derivation of equilibria:

“We must look at the price system such as a mechanism for communicating information if we want to understand its real function [. . .] The most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action” (Hayek 1945)
Allocating scarce resources

- Aggregating all individual demand functions results in the equilibrium price. The Walrasian equilibrium is characterized by homogeneous information and no uncertainty regarding prices and quantities.

Figure 2.1. Walrasian equilibrium.

De Jong & Rindi, p. 33
Transmitting information

- When information is asymmetric, prices are the main source of information.
- Market participants take into account that their moves may influence the market.
- Traders derive conclusions from observations of market movements.
- Definition of a rational market equilibrium according to De Jong & Rindi:

> If the price is a vehicle of information, the equilibrium price is such that, once agents have observed it, they do not wish to trade again, or in other words, they no longer wish to alter their investment decisions.

Resulting is a structure that differs from Walrasian equilibrium!
Characteristics of rational expectations equilibrium

- The market demand function may not be decreasing. When lower prices are observed, demand may fall rather than rise as expected! Market participants conclude that a fall in price may be result of informed traders' lower valuation.

- Demand and supply curves may influence each other: If supply is increasing and prices fall, market participants may lower their demand due to observing the fall and again with the mechanics of above.

- There may be no equilibrium. If prices would be fully informative, there would be no incentive to acquire information, hindering trading, what then would diminish information content of price, and when trading will be done, information is regained through observations. This cycle leads to the paradox of no equilibrium....
Financial Market Equilibrium with symmetric information

• Before dealing with equilibria under asymmetric information, we consider models with symmetric information for one and for multiple assets. We revisit the Capital Asset Pricing Model (CAPM) as well.

• Consider a one-period model with a risk-neutral agent which allocates between a risky asset yielding a return following a normal distribution and a risk-free asset. The agent is initially endowed already with both assets:

  risk-free asset: \( I_f \rightarrow \text{returns: } 1 + r_f \)

  risky asset: \( I \rightarrow \text{returns: } \tilde{F} \sim N(\mu, \sigma) \)
Financial Market Equilibrium with symmetric information

$X$: demand for risky asset

$p$: price of risky asset

$p_f$: price of risk-free asset (normalized to 1)

$$\max_X E[U(\tilde{w})], \quad \tilde{w} = (I + X)\tilde{F} + (I_f - Xp)(1 + r_f)$$

$$\implies E[u'(\tilde{w})(\tilde{F} - p(1 + r_f))] = 0$$
Financial Market Equilibrium with symmetric information

- Taking dependence into account:

\[ E[u'(\tilde{w})(\tilde{F} - p(1 + r_f))] = E[u'(\tilde{w})]E[\tilde{F} - p(1 + r_f)] + \text{Cov}[u'(\tilde{w}), \tilde{F} - p(1 + r_f)] \]

- Following some calculus, we obtain:

\[ E[u'(\tilde{w})]E[\tilde{F} - p(1 + r_f)] + E[u''(\tilde{w})](l + X)\text{Var}(\tilde{F}) = 0 \]

\[ \implies p = \frac{1}{1 + r_f} \left[ E(\tilde{F}) + \frac{E[u''(\tilde{w})]}{E[u'(\tilde{w})]}(l + X)\text{Var}(\tilde{F}) \right] \]

- This yields the price as a function of the first and second derivatives of the expected utility function, the return of the risk-free rate, the expected return of the risky asset and the variance of its returns, as well of the initial endowments.
Financial Market Equilibrium with symmetric information

- Note that while the return on the risk-free asset is decreasing the equilibrium price, and the expected return of the risky asset is increasing it, the effect of the variance of the returns from the risky asset is dependent on the following term which is related to the absolute risk-aversion of the agent:

$$R_a(\tilde{w}) = -\frac{[u''(\tilde{w})]}{[u'(\tilde{w})]}$$

Note: Here we have to deal with expected values....

- By assumption from utility theory, the denominator is positive, as increasing wealth is increasing utility. For risk-averse individuals, the nominator takes on negative values, resulting in a positive value for the absolute risk-aversion measure. Accordingly, negative values are obtained for risk-lovers, and zero represents risk-neutral individuals.
Financial Market Equilibrium with symmetric information

• Applying an assumption of risk neutrality to the obtained result for the price yields the following:

\[ p = \frac{E(\tilde{F})}{1 + r_f} \]

>>> The equilibrium price equation reduces to a simple ratio of the expected return from the risky asset and a discount factor.

• Consider another case, the special case of constant absolute risk aversion (CARA). For CARA functions, the absolute risk aversion takes on a constant value over all values of wealth or outcomes:

\[ R_a(\tilde{w}) = -\frac{u''(\tilde{w})}{u'(\tilde{w})} = A \implies \frac{u''(\tilde{w})}{u'(\tilde{w})} = -A \]
Financial Market Equilibrium with symmetric information

\[ p = \frac{1}{1 + r_f} \left[ E(\tilde{F}) - A(I + X) \text{Var}(\tilde{F}) \right] \]

- This again yields an interpretable formulation of the equilibrium price in which the nominator is discounted by the risk-free return, only now the nominator is reduced with a risk discount that depends on the variance of the returns of the risky asset, the endowments, and the degree of absolute risk aversion. All variables entering the equation as factors decrease the equilibrium price!

- Defining the expected return as the expected payoff minus the price in equilibrium divided by the price in equilibrium, the obtained result with CARA is:

\[ E(\tilde{r}) = \frac{E(\tilde{F}) - p}{p} = r_f + A(I + X) \text{Var}(\tilde{F}) / p \]
Financial Market Equilibrium with symmetric information

• For the general result we would obtain:

\[
E(\tilde{r}) = \frac{E(\tilde{F}) - p}{p} = r_f - \frac{E[u''(\tilde{w})]}{E[u'(\tilde{w})]} (1 + X) \frac{\text{Var(\tilde{F})}}{p}
\]

• The results show that the return includes a risk premium over the risk-free asset, dependent on the amounts held, the risk aversion and the variance of the payoff of the risky asset.

• For the case of risk-neutrality, the risk premium is no more existent due to the fact that the absolute risk-aversion is zero:

\[
E(\tilde{r}) = \frac{E(\tilde{F}) - p}{p} = r_f
\]
Financial Market Equilibrium with symmetric information

- Extending the framework to \( N \) risky assets yields the following utility function with respect to the vectors

\[
E[U(\tilde{w})] = E[U((\tilde{l} + \tilde{X})\tilde{F} + (I_f - X'\tilde{p})(1 + r_f))]
\]

with \( \tilde{p} = [p_1...p_N]' \), \( \tilde{F} = \tilde{F}_1...\tilde{F}_N]' \) and \( \tilde{l} = [l_1...l_N]' \)

- We obtain the result as follows which is the respective multivariable counterpart of the previous result for one risky asset:

\[
E[u'(\tilde{w})(\tilde{F}_i - p_i(1 + r_f))] = E[u'(\tilde{w})]E[\tilde{F}_i - p_i(1 + r_f)]
+ Cov[u'(\tilde{w}), (\tilde{F}_i - p_i(1 + r_f))]
\]
Financial Market Equilibrium with symmetric information

\[ p_i = \frac{1}{1 + r_f} \left[ E(\tilde{F}_i) + \frac{E[u''(\tilde{w})]}{E[u'(\tilde{w})]} \text{Cov}[\tilde{w}, \tilde{F}_i] \right] \]

- The result of the maximization of utility is analogous to the case of one risky asset, with the equilibrium price depending on the risk-free asset’s payoff, the expected payoffs from the risky assets, the absolute risk-aversion and the covariances of the assets and total wealth. The second term is again interpreted as risk premium demanded in equilibrium.

- As for the single asset model, the expected return follows with the analogous reasoning:

\[ E(\tilde{r}_i) = r_f + \left[ -\frac{E[u''(\tilde{w})]}{E[u'(\tilde{w})]} \right] \text{Cov}[\tilde{w}, \tilde{r}_i] \quad \text{with} \quad \tilde{r}_i = \frac{\tilde{F}_i - p_i}{p_i} \]
Financial Market Equilibrium with symmetric information

- Assuming now that the market is fully described by the quantities found in the equilibrium conditions above:

  market portfolio: \[ \tilde{F}_m = \sum_{i=1}^{N} X_i \tilde{F}_i \]

  price of the market portfolio: \[ p_m = \sum_{i=1}^{N} X_i p_i \]

  number of shares of each asset \( i \): \[ \mu_i = \frac{X_i p_i}{\sum_{i=1}^{N} X_i p_i} \]

  return on asset \( i \) and market portfolio \( m \): \[ \tilde{r}_i = \frac{(\tilde{F}_i - p_i)}{p_i} \quad \tilde{r}_m = \frac{(\tilde{F}_m - p_m)}{p_m} \]
Financial Market Equilibrium with symmetric information

- Multiplying both terms of the expected return equation above by $\frac{\mu_i}{\rho_m}$ and summing across all assets yields:

$$E(\tilde{r}_m) - r_f = \left[ - \frac{E[u''(\tilde{W})]}{E[u'(\tilde{W})]} \right] \text{Cov}[\tilde{W}, \tilde{r}_m]$$

$$E(\tilde{r}_i) - r_f = \frac{\text{Cov}[\tilde{r}_m, \tilde{r}_i]}{\text{Var}(\tilde{r}_m)} [E(\tilde{r}_m) - r_f] = \beta_i [E(\tilde{r}_m) - r_f]$$

- Some transformations based on the covariance relations yields:

$$\implies p_i = \frac{E(\tilde{F}_i)}{1 + r_f + \beta_i [E(\tilde{r}_m) - r_f]}$$

- As is obvious now, we derived exactly what is defined in the pricing equation and the expected return equation in the CAPM. Therefore, the equilibrium price and return formulation in the model of symmetric information is equivalent to the CAPM definitions!
Financial Market Equilibrium with asymmetric information

- So far, we assumed that information is symmetric, i.e. all market participants possess the same information about the market assets.
- Now, we drop that assumption and divide investors into groups, with informed, uninformed and noise traders.
- As information is asymmetric and we have different groups, we have several demand functions. Consider first the one for the noise traders:

\[ \tilde{x} \sim N(0, \sigma^2_x) \]

- Further assumptions are a normalization of the risk-free rate to zero, that there are no endowments, and we have one risky asset.
- We assume that the pay-off of the risky asset is normally distributed, that investors have an initial prior expectation about it and that at time 0, the informed traders receive a signal. The signal is independently and identically distributed:

\[ \tilde{F} \sim N(F, \sigma^2_F) \quad \text{and} \quad \tilde{S|F} \sim N(F, \sigma^2_S) \]
Financial Market Equilibrium with asymmetric information

- Using the rule of Bayes, we obtain the posterior as:

\[
\tilde{F} | S \sim \left( \frac{1}{\sigma_S^2} S + \frac{1}{\sigma_F^2} \bar{F}, \frac{1}{\sigma_F^2 + 1/\sigma_S^2} \right)
\]

- or:

\[
\tilde{F} | S \sim N(\beta S + (1 - \beta) \bar{F}, (1 - \beta) \sigma_F^2)
\]

- where:

\[
\beta = \frac{\tau_S}{\tau_F + \tau_S}, \quad \tau_S = \frac{1}{\sigma_S^2}, \quad \tau_F = \frac{1}{\sigma_F^2}
\]
Financial Market Equilibrium with asymmetric information

- Consider now the following structure:

  \[ t = 0 \to t = 1 \to t = 2 \]

- \( t = 0 \): structure as described
- \( t = 1 \): trading takes place, risk-averse agents submit limit-orders, noise traders submit market orders
- \( t = 2 \): value becomes public information
Assuming CARA for the risk-averse agents of the two groups of informed and uninformed investors, we use the following negative exponential utility function for maximization and profit function:

\[ E[U(\tilde{\pi})] = E[-e^{-A\tilde{\pi}}] \]
where \( A \) is the coefficient of risk aversion and \( \tilde{\pi} \) is the end-of-period profit: \( \tilde{\pi} = X(\tilde{F} - p) \)

As \( \tilde{F} \) is normally distributed, \( \tilde{\pi} \) is also normal:

\[ \tilde{\pi} \sim N\left(X(E(\tilde{F}) - p), X^2 \text{Var}(\tilde{F})\right) \]
Financial Market Equilibrium with asymmetric information

- Applying the properties of the moment generating function yields the following maximization problem and first order condition:

$$\arg\max_{\chi} E[-e^{-A\tilde{\pi}}] = \arg\max_{\chi} \left\{ E(\tilde{\pi}) - \frac{A}{2} \text{Var}(\tilde{\pi}) \right\}$$

$$\chi = \frac{E(\tilde{F}) - p}{A \text{Var}(\tilde{F})}$$

- This result for the demand when considering uninformed traders can be analogously defined for the traders receiving information:

$$\chi_I = \frac{E(\tilde{F}|S) - p}{A \text{Var}(\tilde{F}|S)}$$
Financial Market Equilibrium with asymmetric information

- Having found the results for the demand functions of the respective groups, we can now solve the model, where we divide into two parts:
  - the model with assumption of naive expectations, and
  - the more complex rational expectations model where we will find several possible equilibria depending on the assumptions made
Naive Expectations

- Combining the demand functions to set up the aggregate market-clearing condition, where the demand is submitted to a Walrasian market-clearing auctioneer yields:

\[ N \tilde{X}_i + M \tilde{X} + Z\tilde{x} = 0 \]

\[ N \left( \frac{E(\tilde{F}|S) - p}{A \text{Var}(\tilde{F}|S)} \right) + M \left( \frac{E(\tilde{F}) - p}{A \text{Var}(\tilde{F})} \right) + Z\tilde{x} = 0 \]

- Based on the demand functions and the level of information available, we can derive the equilibrium price as follows:

\[ p = \mu_1 E(\tilde{F}|S) + (1 - \mu_1) E(\tilde{F}) + \mu_2 \tilde{x} \]

with \( \mu_1 = \frac{N \text{Var}(\tilde{F})}{N \text{Var}(\tilde{F}) + M \text{Var}(\tilde{F}|S)} \) and \( \mu_2 = \frac{A Z \text{Var}(\tilde{F}) \text{Var}(\tilde{F}|S)}{N \text{Var}(\tilde{F}) + M \text{Var}(\tilde{F}|S)} \)
Naive Expectations

- Thus, the equilibrium price is a combination of the weighted average of the expected cash flow of insiders, uninformed traders and a risk premium. With the previously found definition of the market posterior $E(\tilde{F} \mid S)$ we see the equilibrium price as follows:

$$p = E(\tilde{F}) + \mu_1 \frac{ Cov(\tilde{F}, \tilde{S})}{Var(\tilde{S})} (\tilde{S} - E(\tilde{S})) + \mu_2 \tilde{x}$$

- This price is a noisy version of the signal, but the equilibrium derived suffers from the problem that uninformed traders only use their previous prior information, called naïve expectation building. Rational expectation building would mean that they use information that is contained in the price they observe.

- In the following, models of the rational expectation type are developed.
Rational Expectations

- We assume that both the signal and an error term are normally distributed (and independent from each other), changing the risky asset’s payoff accordingly:

\[ \tilde{F} = \tilde{S} + \tilde{\varepsilon} \]

- with:

\[ \tilde{S} \sim N(0, \sigma_S^2) \quad \tilde{\varepsilon} \sim N(0, \sigma_{\varepsilon}^2) \quad \tilde{S} \perp \tilde{\varepsilon} \]

- As a result, the distribution of the asset payoff can be written as:

\[ \tilde{F} \sim N(0, \sigma_S^2 + \sigma_{\varepsilon}^2) \]
Rational Expectations

• Again assuming that insiders observe the signal and build their conditional expectations yields the following:

\[ E(\tilde{F}|S) = S \quad \text{and} \quad \text{Var}(\tilde{F}|S) = \sigma^2 \varepsilon \]

• In contrast to before, the market participants do not stick to their prior expectations but update with respect to the market price. This changes the demand functions for both uninformed and informed traders:

\[ \tilde{\chi} = \frac{E(\tilde{F}|p) - p}{A \text{Var}(\tilde{F}|p)} \]

\[ \tilde{\chi}_I = \frac{E(\tilde{F}|S, p) - p}{A \text{Var}(\tilde{F}|S, p)} = \frac{E(\tilde{F}|S) - p}{A \text{Var}(\tilde{F}|S)} \]
Rational Expectations

- A crucial aspect here is that for the insiders the public and private information is observable, and therefore they cannot get more new information on the asset price than the one they received through the signal:

\[ E(\tilde{F}|S, p) = E(\tilde{F}|S) \quad \text{and} \quad \text{Var}(\tilde{F}|S, p) = \text{Var}(\tilde{F}|S) \]

- At this point, rational expectation building comes into effect: When taking into account that prices contain information and that other traders act accordingly, price and signal are correlated. Such an equilibrium is called rational expectations equilibrium.
Rational Expectations

• Based upon several assumptions, one obtains the linear rational expectations model:

• The uniformed agents make a conjecture on the equilibrium price function equal to:

\[ \tilde{p} = \alpha_1 \tilde{S} + \alpha_2 \tilde{x} \]

1. They use this conjecture to estimate \( E(\tilde{F} \mid p) \) and \( \text{Var}(\tilde{F} \mid p) \) so as to determine \( X \).

2. Equilibrium is attained when the price that meets the market-clearing condition \( N X_I + M X + Z \tilde{x} = 0 \) is a linear combination of the signal and the demanded amount, or is equal to the conjecture made. In the latter case, the equilibrium implies that agents' expectations coincide with the outcome.
Rational Expectations

- Using the previously found demand functions and using the assumptions that insiders use the signal, with uninformed investors extracting information from the market price, yields the following:

$$X_I = \frac{S - p}{A \sigma^2 \varepsilon}$$

- Uniformed agents extract information from the market price and hence we have to compute the conditional mean and variance of $\tilde{F}$:

$$E(\tilde{F} | p = \alpha S + \alpha X) = \frac{\alpha_1 \sigma^2_s}{\alpha_1^2 \sigma^2_s + \alpha_2^2 \sigma^2_x} p = \vartheta p$$

$$\text{Var}(\tilde{F} | p = \alpha S + \alpha X) = \sigma^2_s + \sigma^2_\varepsilon - \frac{\alpha_1^2 \sigma^4_s}{\alpha_1^2 \sigma^2_s + \alpha_2^2 \sigma^2_x}$$
Rational Expectations

• Accordingly, the market clearing price is defined as:

\[ \tilde{p} = \varphi_1 \{\alpha_1, \alpha_2\} \tilde{S} + \varphi_2 \{\alpha_1, \alpha_2\} \tilde{x} \]

• with:

\[ \varphi_1 \{\alpha_1, \alpha_2\} = \frac{N \text{Var}(\tilde{F}|p)}{M \sigma^2_{\hat{e}}(1 - \vartheta) + N \text{Var}(\tilde{F}|p)} \]

• and:

\[ \varphi_2 \{\alpha_1, \alpha_2\} = \frac{A \text{Var}(\tilde{F}|p)\sigma^2_{\hat{e}}Z}{M \sigma^2_{\hat{e}} + N \text{Var}(\tilde{F}|p)} \]
Rational Expectations

- Ensuring that the actual price rule is equivalent to the expected price rule from the first assumption of the linear REE, yields:

\[
\alpha_1 = \varphi_1 \{\alpha_1, \alpha_2\} \\
\alpha_2 = \varphi_2 \{\alpha_1, \alpha_2\}
\]

- The solution to this system is a pair of \(\alpha_1^*\) and \(\alpha_2^*\) such that we have the REE price function:

\[
\tilde{p}^* = \alpha_1^* \tilde{S} + \alpha_2^* \tilde{x}
\]

\[
\alpha_1^* = f_1\{A, M, N, Z, \sigma_S^2, \sigma_{\varepsilon}^2, \sigma_x^2\}, \alpha_2^* = f_2\{A, M, N, Z, \sigma_S^2, \sigma_{\varepsilon}^2, \sigma_x^2\}
\]

- This equilibrium is to be calculated in the following, where one is distinguishing between deterministic supply and random noise trading equilibria.
REE: fully revealing equilibrium. (det supply)

- Recall the naïve equilibrium price:

\[ p = \mu_1 E(\tilde{F}|S) + (1 - \mu_1) E(\tilde{F}) + \mu_2 x \]

\[ = \mu_1 \left( \frac{1}{\sigma_S^2} S + \frac{1}{\sigma_F^2} \tilde{F} \right) + (1 - \mu_1) \tilde{F} + \mu_2 x \]

- From this result, the uninformed trader - now assumed to build rational expectations, can straight up derive the signal that the insiders obtain, as the signal is the only variable that was unknown to the group of uninformed market participants.
REE: fully revealing equilibrium (det. supply)

- Therefore, the demand from uninformed traders which and the REE equilibrium price are now conditioned on the price and the signal:

\[ X = X_I = \frac{E(\tilde{F}|S) - p}{A \ Var(\tilde{F}|S)} \]

\[ p = E(\tilde{F}|S) - A \ Var(\tilde{F}|S) Z \frac{X}{N + M} \]

- This result allows for the analogous interpretations as before, with the equilibrium price being a combination of the expected payoff and a risk premium. However, now the signal conditioning due to price observation and thus signal extraction means that this equilibrium is fully revealing with respect to the private information and prices are strong-form efficient: they fully incorporate public and private information.
RE inconvenient: noisy rational expectations equilibrium

- When the assumption of a fixed supply from above is replaced with random noise trading, the uninformed market participants cannot easily derive the signal. However, the assumptions previously defined with the conjecture that is built on prices, can be used to have the uninformed agents building expectations on the demand functions of the informed traders and the uninformed agents themselves.

- Using the demand function $X = \frac{S - \sigma^2}{A \sigma^2}$ of the market participant receiving a signal, the rational uninformed traders know that the insiders do not get information from the market price and thus do not update their demand function. In addition, they make the assumption that all other $M-1$ uninformed traders have a naturally downward-sloping demand function: $X = -Hp$
REE: noisy rational expectations equilibrium

- This results in the following market-clearing condition and price equation:

\[ N \left( \frac{\tilde{S} - p}{A\sigma_\varepsilon^2} \right) - (M - 1) Hp + X + Z \tilde{x} = 0 \]

\[ p = \lambda \left[ \frac{N\tilde{S}}{A\sigma_\varepsilon^2} + X + Z \tilde{x} \right] \quad \text{and} \quad \lambda = \left[ \frac{N}{A\sigma_\varepsilon^2} + (M - 1)H \right]^{-1} \]

- In the following, the signal extraction is done by using the information available and observable, and by building on the assumptions for the linear REE that conjectures on the price and the demand function are coinciding with the realizations. Solving the market-clearing condition after some algebra:

\[ p^* = \left[ \frac{N}{A\sigma_\varepsilon^2} + MH^* \right]^{-1} \left[ \frac{N}{A\sigma_\varepsilon^2} \tilde{S} + Z\tilde{x} \right] \]
REE: noisy rational expectations equilibrium

• In this equilibrium, the realizations are coinciding with the conjectures, which is due to the rational expectations driving the convergence to equilibrium. As noted in De Jong and Rindi, this result is very comfortable with respect to the actual financial market: it is just straightforward to assume that actions of market participants that possess information drive prices and that other market participants observe market movements and make use of the information contained in the prices observed.

• Especially from a quantitative view, the computation of market numbers is interesting, with the following being the indicators for liquidity, volatility and informational efficiency based on the defined equilibrium:
REE: noisy rational expectations equilibrium

\[ L = \left| \frac{dp}{dx} \right|^{-1} = \left[ \frac{N}{A\sigma^2_{\varepsilon}} + MH^* \right] = \left( \frac{N}{A\sigma^2_{\varepsilon}} + M \left( 1 - \frac{\text{Cov}(F, \tilde{\Theta})}{\text{Var}(\tilde{\Theta})} \right) \right) \]

\[ \text{Var}(p^*) = (L)^{-2} \left[ \frac{N^2}{A^2\sigma^4_{\varepsilon}} \sigma^2_S + Z^2 \sigma^2_x \right] \]

\[ IE = \text{Var}(\tilde{F} | \Theta)^{-1} = \left( \frac{\sigma^2_S + \sigma^2_{\varepsilon} - \frac{\sigma^4_S}{A^2\sigma^4_{\varepsilon}Z^2}}{\frac{\sigma^2_S + \frac{\sigma^4_S}{A^2\sigma^4_{\varepsilon}Z^2}}{N^2}} \right) \]

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Financial Market Equilibrium with Asymmetric Information and Strategic agents

- Following the derivations for both symmetric and asymmetric information, the third model type to be considered is extending the asymmetric information structure to allow for strategic agents.
- Strategic in that sense means that market participants take into account how their activities influence the market.
- Assumptions are that informed traders receive a normally distributed initial endowment shock that is increasing or decreasing their endowment. While the endowment shock and the signal are unobservable to uninformed traders, each informed agent has a linear demand function:

\[ X_I^S = \frac{\tilde{S} - p_S - A\sigma^2 I}{A\sigma^2 + \lambda I} = D(\tilde{S} - pS) - G\tilde{I} \]

- with: \( D = \frac{1}{A\sigma^2 + \lambda I} \) and \( G = \frac{A\sigma^2}{A\sigma^2 + \lambda I} \)
Financial Market Equilibrium with Asymmetric Information and Strategic agents

- The new parameter measures the impact that the order of the informed trader has on the market.
- Uninformed traders again extract the signal from the market price and formulate limit-orders.
- Following the usual equating of the first-order conditions and the market-clearing condition, we obtain the solution for the strategic equilibrium price:

$$p^S = [LSA]^{-1} \left[ ND\tilde{S} - NG\tilde{l} + Z\tilde{x} \right]$$

$$LSA = \left| \frac{dp}{dx} \right|^{-1} = [ND + MH^S*]$$

$$= \frac{N}{A\sigma^2 + \lambda I} + M \left( 1 - \frac{Cov(\tilde{F}, \tilde{\Theta}_S)}{Var(\tilde{\Theta}_S)} \right)$$

$$+ M \left( A \frac{Cov(\tilde{F}, \tilde{\Theta}_S)M}{Var(\tilde{\Theta}_S)N} (A\sigma^2 + \lambda I) \right)$$
Financial Market Equilibrium with Asymmetric Information and Strategic agents

- The most crucial difference between the result for competitive agents and strategic agents is the lower liquidity in the model with strategic agents, as their knowledge about the impact of their orders holds them back from trading more aggressively!
Mandatory: F. de Jong and B. Rindi
*The Microstructure of Financial Markets*. Chapter 2

The references in the de Jong and Rindi chapter contain many interesting articles for further reading!