Problem 1

a) The top plot of Figure 1 shows the daily (continuously compounded) percentage returns, $r_t$, of the Japanese Yen against the US Dollar over the period 1978–2003. The bottom plot shows the sample autocorrelation functions (SACF) of the raw (left panel) and squared returns (right panel). The dashed lines represent approximate 95% confidence intervals.

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1 That is, if $S_t$ is the exchange rate at day $t$, $r_t = 100 \times (\log S_t - \log S_{t-1})$. 

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Exercise Session for Financial Data Analysis

Summer term 2010

Problem Set 9
Describe what you observe in Figure 1 *insofar it is typical* for many financial return series, and exemplify the implications of your observations.

Also describe a class of models that may be used to capture the features that you find most pronounced in Figure 1. Explain why this model class may be adequate for modeling these properties.

b) Consider the ARCH(1) process with normal innovations, i.e.,

$$\epsilon_t = \eta_t \sigma_t, \quad \eta_t \overset{iid}{\sim} N(0, 1), \quad \sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2, \quad \alpha_0 > 0, \quad \alpha_1 \geq 0. \quad (1)$$

Assume that $3\alpha_1^2 < 1$. Find the unconditional kurtosis, $\kappa := \mathbb{E}(\epsilon_t^4)/[\mathbb{E}(\epsilon_t^2)^2]$, of the process (1) and show that, if $\alpha_1 > 0$, it exceeds the value of the normal distribution. What is the usual interpretation of $\kappa$ in the empirical finance literature?
Problem 2

Identification of the AR(1)MA processes in Figures 2 and 3.
Problem 3

Consider the following AGARCH(1,1) process, as proposed by Engle (1990):

\[ \epsilon_t = \eta_t \sigma_t, \quad \eta_t \overset{iid}{\sim} \mathcal{N}(0, 1) \]  \hspace{1cm} (2)

\[ \sigma_t^2 = \alpha_0 + \alpha_1 (\epsilon_{t-1} - \theta)^2 + \beta_1 \sigma_{t-1}^2 \]  \hspace{1cm} (3)

\[ \alpha_0 > 0, \quad \alpha_1 \geq 0, \quad \beta_1 \geq 0, \quad \alpha_1 + \beta_1 < 1, \quad \theta \in \mathbb{R}. \]  \hspace{1cm} (4)

For the model defined by (2)–(4), find \( \text{Cov}(\epsilon_t^2, \epsilon_{t-1}) \), i.e., the covariance between \( \epsilon_t^2 \) and \( \epsilon_{t-1} \), and interpret your result.
Problem 3.2

The bilinear GARCH(1,1) (BL–GARCH) model of Storti and Vitale (2003) is defined by

\[ \epsilon_t = \sigma_t z_t, \quad z_t \sim \text{iid } N(0, 1) \]
\[ \sigma_t^2 = a_0 + a_1 \epsilon_{t-1}^2 + b_1 \sigma_{t-1}^2 + c_1 \epsilon_{t-1} \sigma_{t-1}. \]  

(5)

- Explain the role of parameter \( c_1 \) in the variance equation (5). Which sign do you expect for stock returns?

- Show that, for \( a_0, a_1, b_1 > 0 \), \( c_1^2 < 4a_1b_1 \) makes sure that the variance remains positive.

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Problem 4

a) Consider a stationary AR(2) process,

\[ Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t, \quad \epsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2). \quad (6) \]

(i) For the process defined in (6), derive the Yule–Walker equations and compute \( \rho(1) \) and \( \rho(2) \), where \( \rho(\tau) \) is the autocorrelation function of \( \{Y_t\} \) at lag \( \tau \), i.e.,

\[ \rho(\tau) := \text{Corr}(Y_t, Y_{t-\tau}). \]

Also briefly characterize the behavior of \( \rho(\tau) \) for \( \tau \geq 3 \).

(ii) Use the Yule–Walker equations derived in part (i) to construct a simple estimator of the parameters \( \phi_1 \) and \( \phi_2 \) of model (6).

(iii) Without doing any computations, what is the conditional maximum likelihood estimator of the parameters \( \phi_0, \phi_1 \) and \( \phi_2 \) of model (6)?

b) (note that this does not require any computations...) Consider a stationary ARMA(2,2) process,

\[ Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2}, \quad \epsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2). \quad (7) \]

Find the unconditional kurtosis of the process defined by (7).