Problem 1 Consider the ARMA(2,1) process

\[ Y_t = 0.3 + 0.4Y_{t-1} + 0.45Y_{t-2} + \epsilon_t + 0.25\epsilon_{t-1}, \quad \epsilon_t \sim \text{Normal}(0, \sigma^2). \] (1)

(i) Is it stationary?
(ii) Is it invertible?
(iii) Find the mean of \( Y_t \).
(iv) Find the coefficients of the MA(\( \infty \)) representation by the method of matching coefficients.

Problem 2 The second-order autoregressive process,

\[ Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t, \] (2)

where \( \epsilon_t \) is white noise, comes up frequently in applications, so it is of interest to characterize its dynamic behavior as a general function of its parameters \( \phi_1 \) and \( \phi_2 \). Process (2) is stationary (causal) if the roots of \( z^2 - \phi_1 z - \phi_2 = 0 \) are smaller than one in magnitude, i.e.,

\[ \left| \frac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2} \right| < 1. \] (3)

(i) Show that (3) is equivalent to the following set of conditions:

\[ \phi_1 + \phi_2 < 1, \quad \phi_2 - \phi_1 < 1, \quad \text{and} \quad |\phi_2| < 1. \] (4)

Conditions (4) describe the stationarity triangle, i.e., a triangular region in the parameter space leading to a stationary solution of process (2). Draw the triangle and also indicate the part of the stationary region where the roots are complex, so that a cyclical behavior of the resulting time series is expected.
Problem 3 There are various asymmetries in many financial return series, which may be explained by agents reacting differently to positive and negative shocks (news) of the same magnitude. A simple asymmetric moving average (asMA) process is given by \(^1\)

\[
Y_t = \epsilon_t + \theta^+ \epsilon^+_{t-1} + \theta^- \epsilon^-_{t-1},
\]

where \(\theta^+\) and \(\theta^-\) are parameters, \(\epsilon_t \stackrel{iid}{\sim} \text{Normal}(0, 1)\), and

\[
\epsilon^+_{t-1} = \max\{\epsilon_{t-1}, 0\}, \quad \epsilon^-_{t-1} = \min\{\epsilon_{t-1}, 0\},
\]

that is, the MA parameter is given by \(\theta^+\) if \(\epsilon_{t-1} > 0\) and by \(\theta^-\) if \(\epsilon_{t-1} < 0\), so there is an asymmetric response to positive and negative shocks (unexpected price changes).

(i) Find the first-order autocovariance of the asMA process (5).\(^2\) What if \(\theta^+ = -\theta^-\)?

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\(^2\)Note that the mean of \(Y_t\) described by (5) is not equal to zero but given by \(E(\theta^+ \epsilon^+_{t-1} + \theta^- \epsilon^-_{t-1}) = (\theta^+ - \theta^-)/\sqrt{2\pi}\). The concrete form of the mean function is unimportant, however, as it cancels out in the calculation of the autocorrelation.