Exercise Session
Problem Set 1

1.) Suppose that a high school student is preparing to take the Scholastic Assessment Test (SAT). Explain why his or her eventual SAT score is properly viewed as a random variable.

(WO, p.746, Appendix B, Problem 1)

2.) Let $X$ denote the annual salary of university professors in the United States, measured in thousands of dollars. Suppose that the average salary is 52.3, with a standard deviation of 14.6. Find the mean and standard deviation when salary is measured in dollars.

(WO, p.748, Appendix B, Problem 9)

3.) For a randomly selected county in the United States, let $X$ represent the proportion of adults over age 65 who are employed, of the elderly employment rate. Then, $X$ is restricted to a value between zero and one. Suppose that the cumulative distribution function for $X$ is given by $F(x) = 3x^2 - 2x^3$ for $0 \leq x \leq 1$. Find the probability that the elderly employment rate is at least 0.6 (60%).

(WO, p.747, Appendix B, Problem 4)

4.) (Requires calculus) Let $X$ denote the prison sentence, in years, for people convicted of car theft in a particular state in the United States. Suppose that the pdf (density) of $X$ is given by

$$f(x) = \frac{1}{9}x^2, \quad 0 < x < 3.$$ Use integration to find the expected prison sentence.

(WO, p.747, Appendix B, Problem 6)

5.) Much is made of the fact that certain mutual funds outperform the market year after year (that is, the return from holding shares in the mutual fund is higher than the return from holding a portfolio such as the S&P 500). For concreteness, consider a 10-year period and let the population be the 4,170 mutual funds reported in *The Wall Street Journal* on January 1, 1995. By saying that performance relative to the market is random, we mean that each fund has a 50-50 chance of outperforming the market in any year and that performance is independent from year to year.

(i) If performance relative to the market is truly random, what is the probability that any particular fund outperforms the market in all 10 years?
(ii) Find the probability that at least one fund out of 4,170 funds outperforms the market in all 10 years. What do you make of your answer?

(iii) If you have a statistical package that computes binomial probabilities, find the probability that at least five funds outperform the market in all 10 years.

(WO, p.747, Appendix B, Problem 3)

6.) Suppose that a large university, college grade point average, $GPA$, and SAT score, $SAT$, are related by the conditional expectation $E(GPA|SAT) = .70 + .002 \cdot SAT$.

(i) Find the expected $GPA$ when $SAT = 800$. Find $E(GPA|SAT = 1,400)$. Comment on the difference.

(ii) If the average $SAT$ in the university is 1,100, what is the average $GPA$? (Hint: Use Property CE.4.)

(iii) If a student’s SAT score is 1,100, does this mean he or she will have the GPA found in part (ii)? Explain.

(WO, p.748, Appendix B, Problem 10)

7.) Calculate the expected value and the variance of the random variable $x_i$ and for the sample mean of i.i.d. (independent and identically distributed) random variables $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ with $(i = 1, ..., n)$. Assume

a) $x_i$ follows a Normal distribution, i.e. $x_i \sim N(\mu, \sigma^2)$.

b) $x_i$ follows a Bernoulli distribution with $P(x_i = 1) = p$ and $P(x_i = 0) = 1 - p$. 